Consumable Workbooks  Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks in both English and Spanish.

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Study Guide and Intervention Workbook 0-07-890739-X</td>
<td>978-0-07-890739-5</td>
</tr>
<tr>
<td>Homework Practice Workbook 0-07-890740-3</td>
<td>978-0-07-890740-1</td>
</tr>
</tbody>
</table>

Spanish Version

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Homework Practice Workbook 0-07-890744-6</td>
<td>978-0-07-890744-9</td>
</tr>
</tbody>
</table>

Answers for Workbooks  The answers for Chapter 9 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

StudentWorks Plus™  This CD-ROM includes the entire Student Edition test along with the English workbooks listed above.

TeacherWorks Plus™  All of the materials found in this booklet are included for viewing, printing, and editing in this CD-ROM.

These masters contain a Spanish version of Chapter 9 Test Form 2A and Form 2C.
Teacher’s Guide to Using the Chapter 9 Resource Masters

The Chapter 9 Resource Masters includes the core materials needed for Chapter 9. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the TeacherWorks Plus™ CD-ROM.

Chapter Resources

*Student-Built Glossary* (pages 1–2) These masters are a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar. Give this to students before beginning Lesson 9-1. Encourage them to add these pages to their mathematics study notebooks. Remind them to complete the appropriate words as they study each lesson.

*Anticipation Guide* (pages 3–4) This master, presented in both English and Spanish, is a survey used before beginning the chapter to pinpoint what students may or may not know about the concepts in the chapter. Students will revisit this survey after they complete the chapter to see if their perceptions have changed.

Lesson Resources

*Study Guide and Intervention* This master provides vocabulary, key concepts, additional worked-out examples and Check Your Progress exercises to use as a reteaching activity. It can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

*Skills Practice* This master focuses more on the computational nature of the lesson. Use as an additional practice option or as homework for second-day teaching of the lesson.

*Practice* This master closely follows the types of problems found in the Exercises section of the Student Edition and includes word problems. Use as an additional practice option or as homework for second-day teaching of the lesson.

*Word Problem Practice* This master includes additional practice in solving word problems that apply the concepts of the lesson. Use as an additional practice or as homework for second-day teaching of the lesson.

*Enrichment* These activities may extend the concepts of the lesson, offer an historical or multicultural look at the concepts, or widen students’ perspectives on the mathematics they are learning. They are written for use with all levels of students.
Graphing Calculator, or Spreadsheet Activities These activities present ways in which technology can be used with the concepts in some lessons of this chapter. Use as an alternative approach to some concepts or as an integral part of your lesson presentation.

Assessment Options
The assessment masters in the Chapter 9 Resource Masters offer a wide range of assessment tools for formative (monitoring) assessment and summative (final) assessment.

Student Recording Sheet This master corresponds with the standardized test practice at the end of the chapter.

Extended Response Rubric This master provides information for teachers and students on how to assess performance on open-ended questions.

Quizzes Four free-response quizzes offer assessment at appropriate intervals in the chapter.

Mid-Chapter Test This 1-page test provides an option to assess the first half of the chapter. It parallels the timing of the Mid-Chapter Quiz in the Student Edition and includes both multiple-choice and free-response questions.

Vocabulary Test This test is suitable for all students. It includes a list of vocabulary words and 10 questions to assess students’ knowledge of those words. This can also be used in conjunction with one of the leveled chapter tests.

Leveled Chapter Tests
• Form 1 contains multiple-choice questions and is intended for use with below grade level students.
• Forms 2A and 2B contain multiple-choice questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
• Forms 2C and 2D contain free-response questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
• Form 3 is a free-response test for use with above grade level students.

All of the above tests include a free-response Bonus question.

Extended-Response Test Performance assessment tasks are suitable for all students. Sample answers and a scoring rubric are included for evaluation.

Standardized Test Practice These three pages are cumulative in nature. It includes three parts: multiple-choice questions with bubble-in answer format, griddable questions with answer grids, and short-answer free-response questions.

Answers
• The answers for the Anticipation Guide and Lesson Resources are provided as reduced pages.
• Full-size answer keys are provided for the assessment masters.
# 9 Student-Built Glossary

This is an alphabetical list of key vocabulary terms you will learn in Chapter 9. As you study this chapter, complete each term’s definition or description. Remember to add the page number where you found the term. Add these pages to your Pre-Algebra Study Notebook to review vocabulary at the end of the chapter.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition/Description/Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>base</td>
<td></td>
<td></td>
</tr>
<tr>
<td>composite number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>cubic function</td>
<td></td>
<td></td>
</tr>
<tr>
<td>exponent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>exponential function</td>
<td></td>
<td></td>
</tr>
<tr>
<td>factor</td>
<td></td>
<td></td>
</tr>
<tr>
<td>factor tree</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition/Description/Example</td>
</tr>
<tr>
<td>-----------------------</td>
<td>---------------</td>
<td>--------------------------------</td>
</tr>
<tr>
<td>monomial</td>
<td></td>
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<tr>
<td>nonlinear function</td>
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<tr>
<td>parabola</td>
<td></td>
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<tr>
<td>power</td>
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<td></td>
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<tr>
<td>prime factorization</td>
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<tr>
<td>prime number</td>
<td></td>
<td></td>
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<tr>
<td>quadratic function</td>
<td></td>
<td></td>
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<tr>
<td>scientific notation</td>
<td></td>
<td></td>
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<tr>
<td>standard form</td>
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<td></td>
</tr>
</tbody>
</table>
### Anticipation Guide

**Powers and Nonlinear Functions**

**Step 1** Before you begin Chapter 9

1. Read each statement.
2. Decide whether you Agree (A) or Disagree (D) with the statement.
3. Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

<table>
<thead>
<tr>
<th>Statement</th>
<th>STEP 2 A or D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. In the expression $4^3$, the exponent is 4.</td>
<td></td>
</tr>
<tr>
<td>2. To evaluate $5^3$, multiply $5 \cdot 5 \cdot 5$.</td>
<td></td>
</tr>
<tr>
<td>3. When simplifying expressions evaluate the powers last.</td>
<td></td>
</tr>
<tr>
<td>4. A prime number is a number that has exactly two factors, 1 and itself.</td>
<td></td>
</tr>
<tr>
<td>5. Multiply powers with the same base by multiplying their exponents.</td>
<td></td>
</tr>
<tr>
<td>6. $x^{-p}$ is equivalent to $\frac{1}{x^p}$.</td>
<td></td>
</tr>
<tr>
<td>7. The number $23.4 \times 10^6$ is written in scientific notation.</td>
<td></td>
</tr>
<tr>
<td>8. $(ab)^m = ab^m$, for all numbers $a$ and $b$ and any integer $m$.</td>
<td></td>
</tr>
<tr>
<td>9. Nonlinear functions do not have constant rates of change.</td>
<td></td>
</tr>
<tr>
<td>10. The graph of a cubic function is called a parabola.</td>
<td></td>
</tr>
</tbody>
</table>

**Step 2** After you complete Chapter 9

1. Reread each statement and complete the last column by entering an A (Agree) or a D (Disagree).
2. Did any of your opinions about the statements change from the first column?
3. For those statements that you mark with a D, use a separate sheet of paper to explain why you disagree. Use examples, if possible.
## Ejercicios preparatorios
### Potencias y funciones no lineal

#### Paso 1
**Antes de comenzar el Capítulo 9**

- Lee cada enunciado.
- Decide si estás de acuerdo (A) o en desacuerdo (D) con el enunciado.
- Escribe A o D en la primera columna O si no estás seguro(a) de la respuesta, escribe NS (No estoy seguro(a)).

<table>
<thead>
<tr>
<th>PASO 1 A, D o NS</th>
<th>Enunciado</th>
<th>PASO 2 A o D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>En la expresión $4^3$, el exponente es 4.</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>Para evaluar $5^3$, multiplica $5 \cdot 5 \cdot 5$.</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>Cuando reduces expresiones, evalúas las potencias al final.</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>Un número primo es un número que tiene exactamente dos factores, 1 y él mismo.</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>Multiplica potencias con la misma base multiplicando sus exponentes.</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>$x^{-p}$ es equivalente a $\frac{1}{x^p}$</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>El número $23.4 \times 10^6$ está escrito en notación científica.</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>$(ab)^m = ab^m$, para todo número $a$ y $b$ y cualquier entero $m$.</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>Las funciones no lineales no tienen tasas de cambio constantes.</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>La gráfica de un función cúbica se llama parábola.</td>
<td></td>
</tr>
</tbody>
</table>

#### Paso 1
**Después de completar el Capítulo 9**

- Vuelve a leer cada enunciado y completa la última columna con una A o una D.
- ¿Cambió cualquiera de tus opiniones sobre los enunciados de la primera columna?
- En una hoja de papel aparte, escribe un ejemplo de por qué estás en desacuerdo con los enunciados que marcaste con una D.


9-1 Study Guide and Intervention

Powers and Exponents

Use Exponents  A number that is expressed using an exponent is called a **power**. The **base** is the number that is multiplied. The **exponent** tells how many times the base is used as a factor. So, \(4^3\) has a base of 4 and an exponent of 3, and \(4^3 = 4 \cdot 4 \cdot 4 = 64\).

\[
\text{base} \rightarrow 4^3 \Leftarrow \text{exponent}
\]

Any number, except 0, raised to the zero power is defined to be 1.

\[
1^0 = 1 \quad 2^0 = 1 \quad 3^0 = 1 \quad 4^0 = 1 \quad 5^0 = 1 \quad x^0 = 1, \ x \neq 0
\]

**Example**  Write each expression using exponents.

a. \(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10\)
   The base is 10. It is a factor 5 times, so the exponent is 5.
   \[10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^5\]

b. \((-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9)\)
   The base is 9. It is a factor 6 times, so the exponent is 6.
   \[(-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9) = (-9)^6\]

c. \((p + 2)(p + 2)(p + 2)\)
   The base is \(p + 2\). It is a factor 3 times, so the exponent is 3.
   \[(p + 2)(p + 2)(p + 2) = (p + 2)^3\]

**Exercises**

Write each expression using exponents.

1. \(5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5\)
2. \((-7)(-7)(-7)\)
3. \(4 \cdot 4\)
4. \(8 \cdot 8 \cdot 8\)
5. \((-2) \cdot (-2) \cdot (-2) \cdot (-2)\)
6. \(\left(\frac{1}{3}\right) \cdot \left(\frac{1}{3}\right) \cdot \left(\frac{1}{3}\right) \cdot \left(\frac{1}{3}\right)\)
7. \((0.4)(0.4)(0.4)\)
8. \(d \cdot d \cdot d \cdot d\)
9. \(m \cdot m \cdot m \cdot m \cdot m \cdot m \cdot m \cdot m\)
10. \(x \cdot x \cdot y \cdot y\)
11. \((z - 4)(z - 4)\)
12. \(3(-t)(-t)(-t)\)
Powers and Exponents

Evaluate Expressions  When evaluating expressions with exponents you must follow the order of operations.

Order of Operations
1. Simplify expressions inside grouping symbols.
2. Evaluate all powers.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

Example 1  ART  An artist is painting a mural that will look like a quilt square. The mural will have an area of $8^2$ square feet. How many square feet is this?

$8^2 = 8 \cdot 8$
$= 64$

The area of the mural will be 64 square feet.

Example 2  Evaluate $x^2 - 4$ if $x = -6$.

$x^2 - 4 = (-6)^2 - 4$
$= (-6)(-6) - 4$
$= 36 - 4$
$= 32$

Exercises

Evaluate each expression.

1. $7^3$

2. $3^6$

3. $(-6)^3$

4. $\left(\frac{1}{5}\right)^4$

5. $(-4)^5$

6. $2^8$

7. $3^3 \cdot 6$

8. $8^3 \cdot 9$

9. $7^2 \cdot 5$

10. $4^2 \cdot 5^2$

11. $(-3)^2 \cdot (-2)^3$

12. $8^2 \cdot 6^3$

Evaluate each expression if $g = 3$, $h = -1$, and $m = 9$.

13. $g^5$

14. $5g^2$

15. $g^2 - m$

16. $hm^2$

17. $g^3 + 2h$

18. $m + hg^3$

19. $4(2m - 3)^2$

20. $-2(g^3 + 1)$

21. $5(h^4 - m^2)$
Write each expression using exponents.

1. \(7 \cdot 7\)
2. \((-3)(-3)(-3)(-3)(-3)\)
3. 4
4. \((k \cdot k)(k \cdot k)(k \cdot k)\)
5. \(p \cdot p \cdot p \cdot p \cdot p \cdot p\)
6. \(3 \cdot 3\)
7. \((-a)(-a)(-a)(-a)\)
8. \(6 \cdot 6 \cdot 6 \cdot 6\)
9. 9 \(\cdot 9 \cdot 9\)
10. \(4 \cdot y \cdot z \cdot z \cdot z\)
11. \(r \cdot r \cdot r \cdot r \cdot t \cdot u \cdot u\)
12. \(5 \cdot 5 \cdot 5 \cdot q \cdot q\)
13. \(8 \cdot 8 \cdot c \cdot c \cdot c \cdot c \cdot d \cdot d \cdot d \)
14. \((-w)(-w)(v)(v)(v)(v)(v)\)
15. \(b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b\)
16. \(10 \cdot 10 \cdot 10 \cdot (-2) \cdot (-2) \cdot (-2) \cdot m \cdot m \cdot m\)

Evaluate each expression if \(a = -3, b = 8,\) and \(c = 2.\)

17. \(4^c\)
18. \(c^0\)
19. \(b^3\)
20. \(c^3 \cdot 3^c\)
21. \(3^c\)
22. \(c^4\)
23. \(c^2 + a\)
24. \(2b^2\)
25. \(b^2 + c^3\)
26. \(a^2\)
27. \(a^3\)
28. \(b^2 + a^3\)
29. \(b^2a\)
30. \((b - c)^2\)
Write each expression using exponents.

1. \(11 \cdot 11 \cdot 11\)  
2. \(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2\)  
3. \(5\)  
4. \((-4)(-4)\)  
5. \(a \cdot a \cdot a \cdot a\)  
6. \(n \cdot n \cdot n \cdot n \cdot n\)  
7. \(4 \cdot 4 \cdot 4\)  
8. \((b \cdot b)(b \cdot b)(b \cdot b)\)  
9. \((-v)(-v)(-v)(-v)\)  
10. \(x \cdot x \cdot z \cdot z \cdot z\)  
11. \(2 \cdot 2 \cdot 2 \cdot 2 \cdot t \cdot t\)  
12. \(m \cdot m \cdot m \cdot n \cdot p \cdot p\)  
13. \((-6)(-6)(-d)(-d)(-d)(-d)\)  
14. \(3 \cdot 3 \cdot 3 \cdot p \cdot q \cdot q \cdot q\)

Evaluate each expression if \(x = 3\), \(y = -2\), and \(z = 4\).

15. \(y^x\)  
16. \(x^c\)  
17. \(y^x\)  
18. \(51^6\)  
19. \(z^2\)  
20. \(x^2\)  
21. \(9^x\)  
22. \(z^2 \cdot 2^2\)  
23. \(y^5\)  
24. \(z^2 - y^4\)  
25. \(x^2 + y^2 + z^2\)  
26. \(z^2 - x^2\)

**FAMILY TREE** For Exercises 27 and 28, refer to the following information.

When examining a family tree, the branches are many. You are generation “now.” One generation ago, your 2 parents were born. Two generations ago, your 4 grandparents were born.

27. How many great-grandparents were born three generations ago?
28. How many “great” grandparents were born ten generations ago?
1. **GEOMETRY** Mr. Daniels is building a clubhouse for his children. He has decided that the floor will be a square with an area of 64 square feet. Write this number using a power greater than 1 and a lesser base.

2. **STOCK MARKET** The Nikkei 225 is a stock market index that records the progress of 225 Japanese companies. Write this number using a power greater than 1 and a lesser base.

3. **NUMBER SENSE** A googol is a very large number expressed as $10^{100}$. Ms. Rogers asked her students to determine which number is larger, a googol or $100^{10}$. Explain how her students might use the idea of repeated factors in order to find the solution.

4. **LIFE SCIENCE** A scientist is studying bacterial growth in the laboratory. She starts her experiment with 1 bacterium and then counts the bacteria at regular intervals and records them in the table below. If the pattern continues, how long will it take to have over 1000 bacteria?

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of cells</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

5. **GEOMETRY** The sides of right triangles have a special relationship. The longest side of a right triangle, always located opposite the right angle, is related to the shorter side lengths by the formula $c = \sqrt{a^2 + b^2}$, where $c$ is the length of the longest side and $a$ and $b$ are the lengths of the sides that intersect to form the right angle.

   a. The following diagram shows a ladder leaning against a wall. The bottom of the ladder is 5 feet from the base of the wall, and the ladder reaches 12 feet up the wall. Find the length of the ladder.

   ![Diagram of a ladder leaning against a wall]

   b. Paula exercises regularly by power walking around a rectangular field. She usually begins at one corner of the field and walks the full perimeter. One day, she takes a shortcut home by walking across the diagonal of the field. How far does she walk across the field?

   ![Diagram of a rectangular field with a diagonal line indicating the shortcut]
Exponents

Numbers can be expressed in several ways. Some numbers are expressed as sums. Some numbers are expressed as products of factors, while other numbers are expressed as powers.

Two ways to express 27 are $3 \cdot 3 \cdot 3$ and $3^3$.

The number 1 million can be expressed in the following ways.

$$1,000,000 = 1000 \cdot 1000 = 100 \cdot 100 \cdot 100 = 10^2 \cdot 10^2 \cdot 10^2$$

$1,000,000 = 1000^2 = 100^3 = 10^6$

Write names for each number below using the given exponents.

1. 16; exponents: 2 and 4
2. 81; exponents: 2 and 4
3. 64; exponents: 2 and 6
4. 256; exponents: 2 and 8
5. 625; exponents: 2 and 4
6. 729; exponents: 2 and 6
7. 2401; exponents: 2 and 4
8. 4096; exponents: 2 and 12
9. 6561; exponents: 2 and 8
10. 39,0625; exponents: 2 and 8

Numbers that can be named as powers with like bases can be multiplied by adding the exponents.

$$8 \cdot 8 = 2^3 \cdot 2^3$$
$$= 2^{3+3}$$
$$= 2^6$$

Write the product of each pair of factors in exponential form.

11. $9 \cdot 9$
12. $4 \cdot 4$
13. $16 \cdot 8$
14. $125 \cdot 25$
15. $27 \cdot 9$
16. $81 \cdot 27$
17. $49 \cdot 49$
18. $121 \cdot 121$
There are several ways to evaluate expressions containing exponents. A base may be raised to any power using $\wedge$, the exponent key. Squares can be evaluated using $x^2$. And cubes can be evaluated using the cube command, which is item 3 in the MATH menu.

Evaluate each expression if $a = 3$, $b = 4$, and $c = -5$.

Store the value of each variable.

Keystrokes: $3 \text{ STO} \alpha[A] \alpha[B] \alpha[C]$.

a. $a^2b^3c$

Keystrokes: $\alpha[A] x^2 \alpha[B] \alpha[C]$.

$b^4 - a^2 - c^5 = 3372$

Keystrokes: $\alpha[B] \wedge 4 - \alpha[A] x^2 - \alpha[C]$.

$c. 2(3b + c)^3$

Keystrokes: $2 (3 \alpha[B] + \alpha[C]) \text{ MATH} 3$.

$2(3b + c)^3 = 686$

**Exercises**

Evaluate each expression.

1. $(-15)^2$

2. $-4^4$

3. $6^5$

4. $9^3$

Evaluate each expression if $x = -2$, $y = 6$, and $z = -3$.

5. $x^4y^2z^2$

6. $z^2 - x^4 + y^3$

7. $12x^2 - x + 8$

8. $12(4y - 5xz)^3$
9-2 Study Guide and Intervention

Prime Factorization

Write Prime Factorizations A prime number is a whole number that has exactly two unique factors, 1 and itself. A composite number is a whole number that has more than two factors. Zero and 1 are neither prime nor composite.

Example 1 Determine whether each number is prime or composite.

a. 29
The only factors of 29 are 1 and 29, so 29 is a prime number.

b. 39
Find the factors of 39 by listing whole number pairs whose product is 39.

\[
39 \times 1 = 39 \\
13 \times 3 = 39
\]

The factors of 39 are 1, 3, 13, and 39. Since the number has more than two factors, it is a composite number.

Any composite number can be written as a product of prime numbers. A factor tree can be used to find the prime factorization.

To make a factor tree:
1. Write the number that you are factoring at the top.
2. Choose any pair of whole number factors of the number.
3. Continue to factor any number that is not prime.

Example 2 Find the prime factorization of 48.

\[
\begin{align*}
48 & \quad 48 \text{ is the number to be factored.} \\
6 \times 8 & \quad \text{Find any pair of whole number factors of 48.} \\
2 \times 3 & \times 2 \times 4 \quad \text{Continue to factor any number that is not prime.} \\
2 \times 2 & \times 2 \times 3 \quad \text{The factor tree is complete when there is a row of prime numbers.}
\end{align*}
\]

The prime factorization of 48 is \(2 \times 2 \times 2 \times 3\) or \(2^4 \times 3\).

Exercises

Determine whether each number is prime or composite.

1. 27  
2. 151  
3. 77  
4. 25

5. 92  
6. 49  
7. 101  
8. 81

Write the prime factorization of each number. Use exponents for repeated factors.

9. 16  
10. 45  
11. 78  
12. 70

13. 50  
14. 102  
15. 76  
16. 56
9-2 Study Guide and Intervention (continued)

Prime Factorization

Factor Monomials Monomials are numbers, variables, or products of numbers and/or variables. Examples of monomials and non-monomials are given below.

<table>
<thead>
<tr>
<th>Monomials</th>
<th>Not Monomials</th>
</tr>
</thead>
<tbody>
<tr>
<td>$38m, 4, r$</td>
<td>$38m + 5, 4 - x, r^2 - s^2$</td>
</tr>
</tbody>
</table>

In algebra, monomials can be factored as a product of prime numbers and variables with no exponent greater than 1. So, $8x^2$ factors as $2 \cdot 2 \cdot 2 \cdot x \cdot x$. Negative coefficients can be factored using $-1$ as a factor.

Example Factor each monomial.

a. $3g^3h^2$

$3g^3h^2 = 3 \cdot g \cdot g \cdot g \cdot h \cdot h$

$g^3 = g \cdot g \cdot g; h^2 = h \cdot h$

b. $-12b^3c^4$

$-12b^3c^4 = -1 \cdot 2 \cdot 2 \cdot 3 \cdot b^3 \cdot c^4$

$-12 = -1 \cdot 2 \cdot 2 \cdot 3$

$= -1 \cdot 2 \cdot 2 \cdot 3 \cdot b \cdot b \cdot b \cdot c \cdot c \cdot c \cdot c$

$= b^3 \cdot c^4 = b \cdot b \cdot b \cdot c \cdot c \cdot c \cdot c$

Exercises

Factor each monomial.

1. $21t$
2. $36xy$
3. $-45c^2$
4. $13b^4$
5. $6m^3$
6. $-20xy^2$
7. $a^2b^2c^3$
8. $25h$
9. $-6f^3g^3$
10. $100k^2l$
11. $-80s^4t^2$
12. $46p^3q^5$
13. $t^2u^3v^4$
14. $24ab^2c^4$
15. $-35x^3y^3$
16. $16r^2s^2t^2$
9-2 Skills Practice

Prime Factorization

Determine whether each number is prime or composite.

1. 41  
2. 29  
3. 87  
4. 36  
5. 57  
6. 61  
7. 71  
8. 103  
9. 39  
10. 91  
11. 47  
12. 67

Write the prime factorization of each number. Use exponents for repeated factors.

13. 20  
14. 40  
15. 32  
16. 44  
17. 90  
18. 121  
19. 46  
20. 30  
21. 65  
22. 80

Factor each monomial.

23. 15t  
24. 16r^2  
25. \(-11m^2\)  
26. \(-49y^3\)  
27. 21ab  
28. \(-42xyz\)  
29. 45j^2k  
30. 17u^2v^2  
31. 27d^4  
32. \(-16cd^2\)
9-2 Practice

Prime Factorization

Determine whether each number is prime or composite.

1. 11  
2. 63  
3. 73  
4. 75  
5. 49  
6. 69  
7. 53  
8. 83

Write the prime factorization of each number. Use exponents for repeated factors.

9. 33  
10. 24  
11. 72  
12. 276  
13. 85  
14. 1024  
15. 95  
16. 200  
17. 243  
18. 735

Factor each monomial.

19. 35v  
20. 49c^2  
21. −14b^3  
22. −81h^2  
23. 33wz  
24. −56ghj

25. NUMBER THEORY Twin primes are a pair of consecutive odd primes, which differ by 2. For example, 3 and 5 are twin primes. Find the twin primes less than 100. (Hint: There are 8 pairs of twins less than 100.)
9-2 Word Problem Practice

Prime Factorization

1. ADDRESSES On the block where Liza lives, the house numbers of all the two-story homes are prime numbers. Liza’s block contains house numbers between 40 and 60. List all of the house numbers of the houses that you know have two stories.

2. PACKAGING Renata’s tool cabinet is a rectangular box with a volume of 105 cubic feet. Find the dimensions (length, width, and height) of the cabinet if each dimension is a whole number greater than 1.

3. CLASS PROJECT Amad and Lynette were researching prime numbers for a class project. Each discovered an expression relating to prime numbers. Amad found that all prime numbers greater than 2 can be expressed as $4a \pm 1$ where $a$ is a natural number. Lynette found that all primes greater than 3 can be expressed as $6b \pm 1$ where $b$ is a natural number. Find the values of $a$ and $b$ for the prime number 113.

4. LANDSCAPING A tiled area next to Gerard’s house has an area of $6rs$.

   Width

   Length

Write two possible pairs of expressions that could represent the length and the width of the tiled area.

5. BIRTHDAY Darnell was born in October. Rather than tell his friends on what day he was born, he tells them that his birthday has three distinct prime factors. On what day was Darnell born?

6. NUMBER THEORY In order to determine if a number is prime or composite, it is helpful to test whether prime numbers are factors of the number, starting with the lowest prime number.

   David wants to determine whether or not the number 73 is a prime number. He has already determined that the prime numbers 2, 3, 5, and 7 are not factors of 73.

   a. How does David know that 4 and 6 cannot be factors of 73?

   b. Does David need to test any other numbers to see if 73 is a prime number? (Hint: $7 \times ? = 73$.)

   c. Use this method to determine if 437 is a prime number. List all the potential factors you tested.
**Prime Pyramid**

A prime number is a whole number that has exactly two factors—itself and 1. The pyramid below is called a prime pyramid. Each row begins with 1 and ends with the number of that row. So, row 2 begins with 1 and ends with 2, row 3 begins with 1 and ends with 3, and so on. In each row, the numbers from 1 to the row number are arranged such that the sum of any two adjacent numbers is a prime number.

For example, look at row 4:
- It must contain the numbers 1, 2, 3, and 4.
- It must begin with 1 and end with 4.
- The sum of adjacent pairs must be a prime number:
  \[1 + 2 = 3, \ 2 + 3 = 5, \ 3 + 4 = 7\]

```
   1  2
  1  2  3
 1  2  3  4
1  4  3  2  5
1  4  3  2  5  6
1  ___ ___ ___ ___ ___ ___ 7
1  ___ ___ ___ ___ ___ ___ 5 ___ 8
1  2  3  4  7  6  5  8  9
1  ___ ___ ___ ___ ___ ___ ___ ___ 10
1  ___ ___ ___ ___ ___ ___ ___ ___ ___ 11
1  ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ 12
```

1. Complete the pyramid by filling in the missing numbers.

2. Extend the pyramid to row 13.

3. Explain the patterns you see in the completed pyramid.
Study Guide and Intervention

Multiplying and Dividing Monomials

Multiply Monomials

When multiplying powers with the same base, add the exponents.

<table>
<thead>
<tr>
<th>Symbols</th>
<th>$a^m \cdot a^n = a^{m+n}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td>$4^2 \cdot 4^5 = 4^{2+5}$ or $4^7$</td>
</tr>
</tbody>
</table>

Example 1

Find each product.

a. $5^7 \cdot 5$

$5^7 \cdot 5 = 5^7 \cdot 5^1$

$= 5^{7+1}$

$= 5^8$

5 = 5

Product of Powers Property; the common base is 5.

Add the exponents.

b. $7^3 \cdot 7^2$

$7^3 \cdot 7^2 = 7^{3+2}$

$= 7^5$

Product of Powers Property; the common base is 7.

Add the exponents.

Example 2

Find each product.

a. $g^3 \cdot g^6$

$g^3 \cdot g^6 = g^{3+6}$

$= g^9$

Product of Powers Property; the common base is $g$.

Add the exponents.

b. $2a^2 \cdot 3a$

$2a^2 \cdot 3a = 2 \cdot 3 \cdot a^2 \cdot a$

$= 2 \cdot 3 \cdot a^{2+1}$

$= 2 \cdot 3 \cdot a^3$

$= 6a^3$

Commutative Property of Multiplication

Product of Powers Property; the common base is $a$.

Add the exponents.

Multiply.

Exercises

Find each product. Express using exponents.

1. $4^7 \cdot 4^6$

2. $v^5 \cdot v^4$

3. $(f^3)(f^9)$

4. $(-31^4)(-31^2)$

5. $(-cr^5)(-r^2)$

6. $22^5 \cdot 22^5$

7. $7h(5h^3)$

8. $-10x^2(7x^3)$

9. $5p^3 \cdot (-4p)$

10. $3d^3 \cdot 12d^3$

11. $(-14x) \cdot x$

12. $9z^3 \cdot 2z \cdot (-z^4)$

13. $3^8 \cdot 3^3$

14. $-7u^6(-6u^5)$

15. $-5m^3(4m^6)$
Divide Monomials When dividing powers with the same base, subtract the exponents.

<table>
<thead>
<tr>
<th>Symbols</th>
<th>$\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td>$\frac{5^6}{5^2} = 5^{6-2}$ or $5^4$</td>
</tr>
</tbody>
</table>

Example 1

Find each quotient.

a. $\frac{(-8)^4}{(-8)^2}$

$$\frac{(-8)^4}{(-8)^2} = (-8)^{4-2}$$

Quotient of Powers Property; the common base is $(-8)$.

$$= (-8)^2$$

Subtract the exponents.

b. $\frac{a^7}{a^3}$

$$\frac{a^7}{a^3} = a^{7-3}$$

Quotient of Powers Property; the common base is $a$.

$$= a^4$$

Subtract the exponents.

Example 2

RIVERS The Mississippi River is approximately $3^7$ miles long. The Kentucky River is approximately $3^5$ miles long. About how many times as long is the Mississippi River than the Kentucky River?

Write a division expression to compare the lengths.

$$\frac{3^7}{3^5} = 3^{7-5}$$

Quotient of Powers Property

$$= 3^2$$ or 9

Subtract the exponents. Simplify.

So, the Mississippi River is approximately 9 times as long as the Kentucky River.

Exercises

Find each quotient. Express using exponents.

1. $\frac{7^5}{7^2}$
2. $\frac{1^8}{1^6}$
3. $\frac{(-12)^3}{(-12)^3}$
4. $\frac{c^{20}}{c^{13}}$
5. $\frac{(-p^{18})}{(-p^{12})}$
6. $\frac{2w^3}{2w}$
7. $\frac{e^{10}}{e^3}$
8. $\frac{k^9}{k}$
9. $3v^3 ÷ 3v$
10. $12x^6 ÷ 12x^2$
11. $(-2a^5) ÷ (-2a)$
12. $5y^8 ÷ 5y^3$
9-3 Skills Practice

Multiplying and Dividing Monomials

Find each product or quotient. Express using exponents.

1. \(2^3 \cdot 2^5\)  
2. \(10^2 \cdot 10^7\)

3. \(1^4 \cdot 1\)  
4. \(6^3 \cdot 6^3\)

5. \((-3)^2(-3)^3\)  
6. \((-9)^2(-9)^2\)

7. \(a^2 \cdot a^3\)  
8. \(n^8 \cdot n^3\)

9. \((p^4)(p^4)\)  
10. \((z^6)(z^7)\)

11. \((6b^3)(3b^4)\)  
12. \((-v)^3(-v)^7\)

13. \(11a^2 \cdot 3a^6\)  
14. \(10t^2 \cdot 4t^{10}\)

15. \((8c^2)(9c)\)  
16. \((4f^8)(5f^6)\)

17. \(\frac{5^{10}}{5^2}\)  
18. \(\frac{10^6}{10^2}\)

19. \(\frac{7^9}{7^6}\)  
20. \(\frac{12^8}{12^3}\)

21. \(\frac{100^9}{100^8}\)  
22. \(\frac{(-2)^3}{-2}\)

23. \(\frac{r^8}{r^2}\)  
24. \(\frac{z^{10}}{z^8}\)

25. \(\frac{q^8}{q^4}\)  
26. \(\frac{g^{12}}{g^8}\)

27. \(\frac{(-y)^7}{(-y)^2}\)  
28. \(\frac{(-z)^{12}}{(-z)^5}\)

29. the product of two squared and two to the sixth power

30. the quotient of ten to the seventh power and ten cubed

31. the product of \(y\) squared and \(y\) cubed

32. the quotient of \(a\) to the twentieth power and \(a\) to the tenth power
9-3 Practice

Multiplying and Dividing Monomials

Find each product or quotient. Express using exponents.

1. $4^2 \cdot 4^3$

2. $9^8 \cdot 9^6$

3. $7^4 \cdot 7^2$

4. $13^2 \cdot 13^4$

5. $(-8)^5(-8)^3$

6. $(-21)^9(-21)^5$

7. $t^9 \cdot t^3$

8. $h^4 \cdot h^{13}$

9. $(m^6)(m^6)$

10. $(u^{11})(u^{10})$

11. $(-r)^7(-r)^{20}$

12. $(-w)(-w)^9$

13. $4d^5 \cdot 8d^6$

14. $7j^{50} \cdot 6j^{50}$

15. $-5b^9 \cdot 6b^2$

16. $12^1 \cdot 12^2$

17. $\frac{6^{11}}{6^3}$

18. $\frac{15^3}{15^2}$

19. $\frac{9^9}{9^7}$

20. $\frac{18^4}{18^3}$

21. $\frac{(-7)^6}{(-7)^5}$

22. $\frac{95^{21}}{95^{18}}$

23. $\frac{n^{30}}{n^{20}}$

24. $\frac{n^{19}}{n^{11}}$

25. the product of five cubed and five to the fourth power

26. the quotient of eighteen to the ninth power and eighteen squared

27. the product of $z$ cubed and $z$ cubed

28. the quotient of $x$ to the fifth power and $x$ cubed

29. SOUND Decibels are units used to measure sound. The softest sound that can be heard is rated as 0 decibels (or a relative loudness of 1). Ordinary conversation is rated at about 60 decibels (or a relative loudness of $10^6$). A rock concert is rated at about 120 decibels (or a relative loudness of $10^{12}$). How many times greater is the relative loudness of a rock concert than the relative loudness of ordinary conversation?
1. **BIOLOGY** Ms. Masse’s biology class is conducting an experiment to record the growth of a certain kind of bacteria. Each student has a lab dish containing 2 bacteria which are able to double every day. How many bacteria will be present in a student’s lab dish after two weeks?

2. **COMPUTERS** In 1995, the average home computer had a speed of about \(10^6\) cycles per second. In 2004, the average home computer had a speed of about \(10^9\) cycles per second. How many times faster were the computers in 2004 as compared to those in 1995?

3. **CATERING** A gourmet meal catering company is planning an event for \(2^7\) people. One week before the event, they find out that the number of people has doubled. Will there be \(2^8\) or \(2^{14}\) people at the event? Explain.

4. **HOMEWORK** Vance and Ko are trying to simplify the expression \(3^8 \times 3^7\). Their answers are different:

   - Ko’s work: \(3^8 \times 3^7 = (3 \times 3)^{8+7} = 9^{15}\)
   - Vance’s work: \(3^8 \times 3^7 = 3^{8+7} = 3^{15}\)

   Which student is correct? Identify the mistake made by the other student.

5. **SOUND** Levels of audible sound are measured in decibels (dB). An increase in 10dB is considered a doubling \((2^1)\) of perceivable sound to the human ear. The table below lists the decibel level of some common sounds.

<table>
<thead>
<tr>
<th>Sound Level (dB)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>Library, no talking</td>
</tr>
<tr>
<td>70</td>
<td>Busy traffic</td>
</tr>
<tr>
<td>80</td>
<td>Hair dryer</td>
</tr>
<tr>
<td>100</td>
<td>Garbage truck</td>
</tr>
<tr>
<td>110</td>
<td>Rock concert</td>
</tr>
<tr>
<td>120</td>
<td>Jet engine</td>
</tr>
<tr>
<td>180</td>
<td>Rocket launching pad</td>
</tr>
</tbody>
</table>

**Source:** Seattle Dept. of Planning and Development

- **a.** Karen was walking on a sidewalk along a road with busy traffic. She noticed that a garbage truck seemed much louder than the traffic. How many times louder was the garbage truck than the busy traffic?

- **b.** Most people do not wear hearing protection when drying their hair. However, airport workers often wear ear protection because of the sound produced by jet engines. How many times louder is a jet engine than a hair dryer?

- **c.** How many times louder is it at a rocket launching pad than in a library?
Dividing Powers with Different Bases

Some powers with different bases can be divided. First, you must be able to write both as powers of the same base. An example is shown below.

\[
\frac{2^5}{8^2} = \frac{2^5}{(2^3)^2} = \frac{2^5}{2^6}
\]

To find the power of a power, multiply the exponents.

\[
= 2^{-1} \text{ or } \frac{1}{2}
\]

This method could not have been used to divide \( \frac{2^5}{9^2} \), since 9 cannot be written as a power of 2 using integers.

Simplify each fraction using the method shown above. Express the solution without exponents.

1. \( \frac{8^2}{2^2} \)
2. \( \frac{16^4}{8^3} \)
3. \( \frac{9^3}{3^5} \)
4. \( \frac{81^4}{3^4} \)
5. \( \frac{9^9}{81^2} \)
6. \( \frac{32^4}{16^4} \)
7. \( \frac{125^2}{25^3} \)
8. \( \frac{6^6}{216^2} \)
9. \( \frac{10^6}{1000^3} \)
10. \( \frac{64^3}{8^5} \)
11. \( \frac{27^5}{9^4} \)
12. \( \frac{343^3}{7^5} \)
9-4 Study Guide and Intervention

**Negative Exponents**

Extending the pattern below shows that $4^{-1} = \frac{1}{4}$ or $\frac{1}{4^1}$.

- $4^2 = 16$
- $4^1 = 4$
- $4^0 = 1$
- $4^{-1} = \frac{1}{4}$

This suggests the following definition.

$a^{-n} = \frac{1}{a^n}$ for $a \neq 0$ and any whole number $n$.  
Example: $6^{-4} = \frac{1}{6^4}$

For $a \neq 0$, $a^0 = 1$.  
Example: $9^0 = 1$

---

**Example 1**

Write each expression using a positive exponent.

a. $3^{-4}$
   
   $3^{-4} = \frac{1}{3^4}$
   
   Definition of negative exponent

b. $y^{-2}$
   
   $y^{-2} = \frac{1}{y^2}$
   
   Definition of negative exponent

---

**Example 2**

Write each fraction as an expression using a negative exponent other than $-1$.

a. $\frac{1}{6^3}$
   
   $\frac{1}{6^3} = 6^{-3}$
   
   Definition of negative exponent

b. $\frac{1}{81}$
   
   $\frac{1}{81} = \frac{1}{9^2}$
   
   Definition of exponent
   
   $= 9^{-2}$
   
   Definition of negative exponent

---

**Exercises**

Write each expression using a positive exponent.

1. $6^{-4}$
2. $(-7)^{-8}$
3. $b^{-6}$
4. $n^{-1}$
5. $(-2)^{-5}$
6. $10^{-3}$
7. $j^{-9}$
8. $a^{-2}$

Write each fraction as an expression using a negative exponent other than $-1$.

9. $\frac{1}{2^2}$
10. $\frac{1}{13^4}$
11. $\frac{1}{25}$
12. $\frac{1}{49}$
13. $\frac{1}{3^3}$
14. $\frac{1}{9^2}$
15. $\frac{1}{121}$
16. $\frac{1}{27}$
Lesson 9-4
Study Guide and Intervention
(continued)

Negative Exponents

Evaluate Expressions  Algebraic expressions with negative exponents can be written using positive exponents and then evaluated.

Example 1  Evaluate $b^{-2}$ if $b = 3$.

\[
b^{-2} = 3^{-2}
\]

Replace $b$ with 3.

\[
= \frac{1}{3^2}
\]

Definition of negative exponent

\[
= \frac{1}{9}
\]

Find $3^2$.

Example 2  Evaluate $8c^{-4}$ if $c = 2$.

\[
8c^{-4} = 8(2)^{-4}
\]

Replace $c$ with 2.

\[
= 8 \cdot \frac{1}{2^4}
\]

Definition of negative exponent

\[
= 8 \cdot \frac{1}{16}
\]

Find $2^4$.

\[
= \frac{8}{16}
\]

Simplify.

\[
= \frac{1}{2}
\]

Simplify.

Exercises

Evaluate each expression if $m = -4$, $n = 1$, and $p = 6$.

1. $p^{-2}$  
2. $m^{-3}$  
3. $(np)^{-1}$  
4. $3^n$

5. $p^m$  
6. $(2m)^{-2}$  
7. $m^{-p}$  
8. $(mp)^{-n}$

9. $4^m$  
10. $-3^{-n}$  
11. $mp^{-2}$  
12. $pm^{-2}$
9-4 Skills Practice

Negative Exponents

Write each expression using a positive exponent.

1. \(3^{-4}\)  
2. \(8^{-7}\)  
3. \(10^{-4}\)

4. \((-2)^{-6}\)  
5. \((-40)^{-3}\)  
6. \((-17)^{-12}\)

7. \(n^{-10}\)  
8. \(b^{-8}\)  
9. \(q^{-5}\)

10. \(m^{-4}\)  
11. \(v^{-11}\)  
12. \(p^{-2}\)

Write each fraction as an expression using a negative exponent other than \(-1\).

13. \(\frac{1}{8^2}\)  
14. \(\frac{1}{10^5}\)  
15. \(\frac{1}{2^3}\)

16. \(\frac{1}{6^7}\)  
17. \(\frac{1}{17^4}\)  
18. \(\frac{1}{21^2}\)

19. \(\frac{1}{3^7}\)  
20. \(\frac{1}{9^2}\)  
21. \(\frac{1}{3^2}\)

22. \(\frac{1}{121}\)  
23. \(\frac{1}{25}\)  
24. \(\frac{1}{36}\)

Evaluate each expression if \(x = 1\), \(y = 2\), and \(z = -3\).

25. \(y^{-z}\)  
26. \(z^{-2}\)  
27. \(x^{-8}\)

28. \(y^{-5}\)  
29. \(z^{-3}\)  
30. \(y^{-1}\)

31. \(z^{-4}\)  
32. \(5^z\)  
33. \(x^{-99}\)

34. \(1^z\)  
35. \(4^z\)  
36. \(y^z\)
9-4 Practice

Negative Exponents

Write each expression using a positive exponent.

1. $7^{-8}$
2. $10^{-6}$
3. $23^{-1}$
4. $(-5)^{-2}$
5. $(-18)^{-10}$
6. $m^{-99}$
7. $(-1)^{-12}$
8. $c^{-6}$
9. $p^{-5}$
10. $g^{-17}$
11. $5z^{-4}$
12. $3t^{-1}$

Write each fraction as an expression using a negative exponent.

13. $\frac{1}{2^{10}}$
14. $\frac{1}{29^3}$
15. $\frac{1}{4^4}$
16. $\frac{1}{39}$
17. $\frac{1}{81^7}$
18. $\frac{1}{m^4}$
19. $\frac{1}{x^3}$
20. $\frac{1}{a^2}$
21. $\frac{1}{49}$
22. $\frac{1}{8}$
23. $\frac{1}{144}$
24. $\frac{1}{169}$

Evaluate each expression if $x = 3$, $y = -2$, and $z = 4$.

25. $x^{-4}$
26. $y^{-2}$
27. $y^{-5}$
28. $z^{-4}$
29. $5^y$
30. $10^y$
31. $3z^{-1}$
32. $z^y$
33. $(xz)^{-2}$

34. HAIR Hair grows at a rate of $\frac{1}{64}$ inch per day. Write this number using negative exponents.
9-4 Word Problem Practice

Negative Exponents

1. SOLAR SYSTEM  The distance between Earth and the Sun is about \( \frac{1}{100,000} \) the diameter of the solar system. Express this number using a negative exponent other than \(-1\).

2. PAPER  The paper used by the students at Hopkins Middle School is approximately \( \frac{1}{216} \) inch thick. Express this number using a negative exponent other than \(-1\).

3. TIME  A microsecond is a measure of time that is equal to one millionth of a second. Express this number as a power of 10 with a negative exponent.

4. MEASUREMENT  There are \( 10^{-2} \) meters in 1 centimeter. At the site of an automobile accident, a state trooper uses a measuring tape to determine that the width of a tire track is 20 centimeters. Express this number as a fraction of a meter in simplest form.

5. HOMEWORK  As Libby was working on her math homework, she computed \( 2^{-3} \) by writing the following equation.

\[
2^{-3} = -8
\]

What was Libby’s error? Explain. Then give the correct answer.

6. INSECTS  Kevin’s father is an entomologist. He studies insects. The table below shows the mass of four common insects.

<table>
<thead>
<tr>
<th>Insect</th>
<th>Mass (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Honeybee</td>
<td>( 8^{-2} )</td>
</tr>
<tr>
<td>Ant</td>
<td>( 16^{-2} )</td>
</tr>
<tr>
<td>Housefly</td>
<td>( 9^{-2} )</td>
</tr>
<tr>
<td>Moth</td>
<td>( 4.5^{-2} )</td>
</tr>
</tbody>
</table>

a. Determine which of these insects weighs the most by first expressing each of the masses in decimal form. Round your answers to the nearest thousandth.

b. How many times heavier is the heaviest insect than the lightest insect? Round your answer to the nearest tenth.

c. What percent greater than a housefly’s mass is the mass of a honeybee? Round your answer to the nearest tenth.

d. It is estimated that an ant can lift approximately 20 times its own body mass. How many grams can the average ant lift? Write your answer as a fraction in simplest form.
Proving Definitions of Exponents

Recall the rules for multiplying and dividing powers with the same base. Use these rules, along with other properties you have learned, to justify each definition. Abbreviations for some properties you may wish to use are listed below.

Associative Property of Multiplication (APM)  Additive Identity Property (AIP)
Multiplicative Identity Property (MIP)  Inverse Property of Addition (IPA)
Inverse Property of Multiplication (IPM)

Write the reason for each statement.

1. Prove: \(a^0 = 1\)

   **Statement**
   Let \(m\) be an integer, and let \(a\) be any nonzero number.

   \[a^m \cdot a^0 = a^{m+0}\]
   \[a^m \cdot a^0 = a^m\]
   \[\frac{1}{a^m} \cdot (a^m \cdot a^0) = \frac{1}{a^m} \cdot a^m\]
   \[\left(\frac{1}{a^m} \cdot a^m\right) \cdot a^0 = \frac{1}{a^m} \cdot a^m\]
   \[1 \cdot a^0 = 1\]
   \[a^0 = 1\]

   **Reason**
   a. Given
   b. 
   c. 
   d. 
   e. 
   f. 
   g. 

2. Prove: \(a^{-n} = \frac{1}{a^n}\)

   **Statement**
   Let \(n\) be an integer, and let \(a\) be any nonzero number.

   \[a^{-n} \cdot a^n = a^{-n+n}\]
   \[a^{-n} \cdot a^n = a^0\]
   \[a^{-n} \cdot a^n = 1\]
   \[(a^{-n} \cdot a^n) \cdot \frac{1}{a^n} = 1 \cdot \frac{1}{a^n}\]
   \[a^{-n} \cdot \left(a^n \cdot \frac{1}{a^n}\right) = 1 \cdot \frac{1}{a^n}\]
   \[a^{-n} \cdot 1 = 1 \cdot \frac{1}{a^n}\]
   \[a^{-n} = \frac{1}{a^n}\]

   **Reason**
   a. Given
   b. 
   c. 
   d. 
   e. 
   f. 
   g. 
   h. 

9-5 Study Guide and Intervention

Scientific Notation

Scientific Notation  Numbers like 5,000,000 and 0.0005 are in standard form because they do not contain exponents. A number is expressed in scientific notation when it is written as a product of a factor and a power of 10. The factor must be greater than or equal to 1 and less than 10.

By definition, a number in scientific notation is written as $a \times 10^n$, where $1 \leq a < 10$ and $n$ is an integer.

Example 1  Express each number in standard form.

a. $6.32 \times 10^5$

$6.32 \times 10^5 = 6.32 \times 100,000$

= $632,000$

$10^5 = 100,000$

Move the decimal point 5 places to the right.

b. $7.8 \times 10^{-6}$

$7.8 \times 10^{-6} = 7.8 \times 0.000001$

= $0.0000078$

$10^{-6} = 0.000001$

Move the decimal point 6 places to the left.

Example 2  Express each number in scientific notation.

a. 62,000,000

To write in scientific notation, place the decimal point after the first nonzero digit, then find the power of 10.

$62,000,000 = 6.2 \times 10,000,000$

= $6.2 \times 10^7$

The decimal point moves 7 places.

The exponent is positive.

b. 0.00025

$0.00025 = 2.5 \times 0.0001$

= $2.5 \times 10^{-4}$

The decimal point moves 4 places.

The exponent is negative.

Exercises

Express each number in standard form.

1. $4.12 \times 10^6$

2. $5.8 \times 10^2$

3. $9.01 \times 10^{-3}$

4. $6.72 \times 10^{-7}$

5. $8.72 \times 10^4$

6. $4.44 \times 10^{-5}$

7. $1.034 \times 10^9$

8. $3.48 \times 10^{-4}$

9. $6.02 \times 10^{-6}$

Express each number in scientific notation.

10. 12,000,000,000

11. 5000

12. 0.00475

13. 0.00007463

14. 235,000

15. 0.000377

16. 7,989,000,000

17. 0.00000403

18. 13,000,000
Scientific Notation

Compare and Order Numbers

You can compare and order numbers in scientific notation without converting them into standard form.

To compare numbers in Scientific Notation, compare the exponents.
- If the exponents are positive, the number with the greatest exponent is the greatest.
- If the exponents are negative, the number with the least exponent is the least.
- If the exponents are the same, compare the factors.

Example 1

Compare each set of numbers using <, > or =.

a. $2.097 \times 10^5$  $\cdot$  $3.12 \times 10^3$
   Compare the exponents: $5 > 3$.
   So, $2.097 \times 10^5 > 3.12 \times 10^3$.

b. $8.706 \times 10^{-5}$  $\cdot$  $8.809 \times 10^{-5}$
   The exponents are the same, so compare the factors: $8.706 < 8.809$.
   So, $8.706 \times 10^{-5} < 8.809 \times 10^{-5}$.

Example 2

ATOMS

The table shows the weight of protons, neutrons, and electrons. Rank the particles in order from heaviest to lightest.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron</td>
<td>$9.109 \times 10^{-31}$</td>
</tr>
<tr>
<td>Proton</td>
<td>$1.672 \times 10^{-27}$</td>
</tr>
<tr>
<td>Neutron</td>
<td>$1.674 \times 10^{-27}$</td>
</tr>
</tbody>
</table>

Step 1: Order the numbers according to their exponents.
The electron has an exponent of $-31$. So, it has the least weight.

Step 2: Order the numbers with the same exponent by comparing the factors.

1.672 < 1.674

So, $1.674 \times 10^{-27} > 1.672 \times 10^{-27} > 9.109 \times 10^{-31}$.
The order from heaviest to lightest is neutron, proton, and electron.

Exercises

Choose the greater number in each pair.

1. $4.9 \times 10^4$, $9.9 \times 10^{-4}$
2. $2.004 \times 10^3$, $2.005 \times 10^{-2}$
3. $3.2 \times 10^2$, 700
4. $0.002$, $3.6 \times 10^{-4}$

Order each set of numbers from least to greatest.

5. $6.9 \times 10^3$, $7.6 \times 10^{-6}$, $7.1 \times 10^3$, $6.8 \times 10^4$
6. $4.02 \times 10^{-8}$, $4.15 \times 10^{-3}$, $4.2 \times 10^2$, $4.0 \times 10^{-8}$
7. $8.16 \times 10^6$, $81,600,000$, $8.06 \times 10^6$, $8.2 \times 10^{-6}$
8. $210,000,000$, $2.05 \times 10^{8}$, $21,500,000$, $2.15 \times 10^6$
9-5 Skills Practice

Scientific Notation

Express each number in standard form.

1. $1.5 \times 10^3$
2. $4.01 \times 10^4$
3. $6.78 \times 10^2$
4. $5.925 \times 10^6$
5. $7.0 \times 10^8$
6. $9.99 \times 10^7$
7. $3.0005 \times 10^5$
8. $2.54 \times 10^5$
9. $1.75 \times 10^4$
10. $1.2 \times 10^{-6}$
11. $7.0 \times 10^{-1}$
12. $6.3 \times 10^{-3}$
13. $5.83 \times 10^{-2}$
14. $8.075 \times 10^{-4}$
15. $1.1 \times 10^{-5}$
16. $7.3458 \times 10^7$

Express each number in scientific notation.

17. $1,000,000$
18. $17,400$
19. $500$
20. $803,000$
21. $0.00027$
22. $5300$
23. $18$
24. $0.125$
25. $17,000,000,000$
26. $0.01$
27. $21,800$
28. $2,450,000$
29. $0.0054$
30. $0.000099$
31. $8,888,800$
32. $0.00912$

Choose the greater number in each pair.

33. $8.8 \times 10^5, 9.1 \times 10^{-4}$
34. $5.01 \times 10^2, 5.02 \times 10^{-1}$
35. $6.4 \times 10^3, 900$
36. $1.9 \times 10^{-2}, 0.02$
37. $2.2 \times 10^{-3}, 2.1 \times 10^2$
38. $8.4 \times 10^2, 839$

Order each set of numbers from least to greatest.

39. $3.6 \times 10^4; 5.8 \times 10^{-3}; 2.1 \times 10^6; 3.5 \times 10^5$

40. $64,000,000; 6.2 \times 10^8; 6,400,000; 6.4 \times 10^6$
9-5 Practice

Scientific Notation

Express each number in standard form.

1. \(2.4 \times 10^4\)
2. \(9.0 \times 10^5\)
3. \(4.385 \times 10^7\)
4. \(1.03 \times 10^8\)
5. \(3.05 \times 10^2\)
6. \(5.11 \times 10^{10}\)
7. \(6.000032 \times 10^6\)
8. \(1.0 \times 10^1\)
9. \(8.75 \times 10^5\)
10. \(8.49 \times 10^{-2}\)
11. \(7.1 \times 10^{-6}\)
12. \(1.0 \times 10^{-3}\)
13. \(4.39 \times 10^{-7}\)
14. \(1.25 \times 10^{-4}\)

Express each number in scientific notation.

15. \(40,000\)
16. \(16\)
17. \(876,000,000\)
18. \(4500\)
19. \(151\)
20. \(0.00037\)
21. \(83,000,000\)
22. \(919,100\)
23. \(5,000,000,000,000\)
24. \(0.13\)
25. \(0.0000007\)
26. \(0.0067\)

Order each set of numbers from least to greatest.

27. \(7.35 \times 10^4, 1.7 \times 10^{-6}, 8.26 \times 10^3, 9.3 \times 10^{-2}\)
28. \(0.00048, 4.37 \times 10^{-4}, 4.02 \times 10^{-3}, 0.04\)

NIAGARA FALLS  For Exercises 29 and 30, use the following information.

Every minute, 840,000,000,000 drops of water flow over Niagara Falls.

29. Write this number in scientific notation.
30. How many drops flow over the falls in a day?
Word Problem Practice

Scientific Notation

1. **EARTH SCIENCE** Mr. Bell’s class is studying the solar system. The circumference of Earth at the equator is about 24,900 miles. Express this number in scientific notation.

2. **LIGHT SPEED** The speed of light is approximately $6.71 \times 10^8$ miles per hour. Express this number in standard form.

3. **EARTH SCIENCE** If it takes light 8.3 minutes to reach the Sun from Earth, use the light speed from Exercise 2 to determine the distance from Earth to the Sun. Write your answer in scientific notation.

4. **EARTH SCIENCE** The students in Mr. Bell’s class have learned that the mass of Earth is approximately $5.97 \times 10^{24}$ kilograms. They have also found that mass of an electron is approximately $9.11 \times 10^{-31}$ kilograms. How many times greater than the mass of an electron is the mass of Earth?

5. **AIRCRAFT** The SR-71 “Blackbird” is one of the world’s fastest airplanes. It is capable of traveling at a cruising speed of Mach 3, or three times the speed of sound. The speed of sound is approximately $7.6 \times 10^2$ miles per hour. What is Mach 3 in miles per hour? Write your answer in scientific notation.

6. **POPULATION** Geographers keep track of how many people live in different areas of the world. They are especially interested in how the populations of certain areas change. The table below shows the population of different regions in 1985 and in 2005.

<table>
<thead>
<tr>
<th>Place</th>
<th>Population</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1985</td>
<td>2005</td>
</tr>
<tr>
<td>Earth</td>
<td>$4.9 \times 10^9$</td>
<td>$6.4 \times 10^9$</td>
</tr>
<tr>
<td>China</td>
<td>$1.1 \times 10^9$</td>
<td>$1.3 \times 10^9$</td>
</tr>
<tr>
<td>India</td>
<td>$7.6 \times 10^8$</td>
<td>$1.1 \times 10^9$</td>
</tr>
<tr>
<td>United States</td>
<td>$2.4 \times 10^8$</td>
<td>$3.0 \times 10^8$</td>
</tr>
</tbody>
</table>

**Source:** U.S. Census Bureau

a. In 2005, how many times greater than China’s population is the population of the world?

b. How many more people inhabited Earth in 2005 than in 1985?

c. What was the percentage increase in population in India from 1985 to 2005? Round your answer to the nearest percent.

d. Was India’s percent increase in population greater than or less than the percent increase of the whole world for the same time period? Explain.
Scientific Notation

It is sometimes necessary to multiply and divide very large or very small numbers using scientific notation.

To multiply numbers in scientific notation, use the following rule.

\[(c \times 10^a)(d \times 10^b) = (c \times d) \times 10^{a+b}.\]

Example 1

\[(3.0 \times 10^4)(-5.0 \times 10^{-2}) = [3.0 \times (-5.0)] \times 10^{4+(-2)}
= -15.0 \times 10^2
= -1.5 \times 10^3 \text{ or } -1500\]

To divide numbers in scientific notation, use the following rule.

\[(c \times 10^a) \div (d \times 10^b) = (c \div d) \times 10^{a-b}.\]

Example 2

\[(24 \times 10^{-4}) \div (1.5 \times 10^2) = (24 \div 1.5) \times 10^{-4-2}
= 16 \times 10^{-6}
= 1.6 \times 10^{-5} \text{ or } 0.000016\]

Exercises

Multiply or divide. Express each product or quotient in scientific notation.

1. \((2.7 \times 10^9) \times (3.1 \times 10^2)\)

2. \((6.1 \times 10^{-2}) \times (1.3 \times 10^5)\)

3. \((5.4 \times 10^{-3}) \div (1.8 \times 10^2)\)

4. \((6.9 \times 10^{-3}) \div (3.0 \times 10^{-8})\)

5. \((1.1 \times 10^{-5}) \times (9.9 \times 10^{-1})\)

6. \((4.0 \times 10^9) \div (1.0 \times 10^{-2})\)

Solve. Write your answers in standard form.

7. ASTRONOMY The distance from Earth to the Moon is about \(2.0 \times 10^5\) miles. The distance from Earth to the Sun is about \(9.3 \times 10^7\) miles. How many times farther is it to the Sun than to the Moon?

8. SALARY If each of the \(3.0 \times 10^4\) people employed by Sunny Motors earned \(4.0 \times 10^4\) dollars last year, how much money did the company pay out to its employees?
Study Guide and Intervention

Powers of Monomials

Power of a Power  You can use the property for finding the product of powers to find a property for finding the power of a power.

\[(h^3)^4 = (h^3)(h^3)(h^3)(h^3)\]
\[= h^3 + h^3 + h^3 + h^3\]
\[= h^{12}\]

The meaning of \((h^3)^4\) is \((h^3)\) should be used as a factor 4 times.

Product of Powers Property

Power of a Power Property
To find the power of a power, multiply the exponents.

\[(a^m)^n = a^{m\cdot n}\]

Example

Simplify.

a. \((4^3)^6\)
\[(4^3)^6 = 4^{3\cdot 6}\]
\[= 4^{18}\]

Power of a Power

Simplify.

b. \((c^2)^7\)
\[(c^2)^7 = c^{2\cdot 7}\]
\[= c^{14}\]

Power of a Power

Simplify.

Exercises

Simplify.

1. \((7^2)^4\)
2. \((12^7)^3\)
3. \((8^5)^7\)
4. \((22^3)^2\)

5. \((x^8)^5\)
6. \((y^2)^8\)
7. \((b^3)^3\)
8. \((r^6)^4\)

9. \((4^3)^{-5}\)
10. \((-6^6)^2\)
11. \((5^3)^{-6}\)
12. \((-10^{10})^{-3}\)

13. \((t^4)^{-2}\)
14. \((-s^4)^9\)
15. \((e^3)^{-6}\)
16. \((d^6)^7\)
9-6 Study Guide and Intervention (continued)

Powers of Monomials

Power of a Product
The Power of a Power Property can be extended to find the power of a product.

\[(3d^2)^3 = (3d^2)(3d^2)(3d^2)\]

The meaning of \((3d^2)^3\) is multiplying \(3d^2\) by itself 3 times.

\[
= 3^3 \cdot (d^2)^3
\]

\[
= 3^3 \cdot (d^2) \cdot (d^2) \cdot (d^2)
\]

The meaning of \((d^2)^3\) is multiplying \(d^2\) by itself 3 times.

\[
= 3^3 \cdot d^{2+2+2}
\]

Product of Powers Property

\[
= 27 \cdot d^6 \text{ or } 27d^6
\]

Power of a Product Property
To find the power of a product, find the power of each factor and multiply.

\[(ab)^m = a^m b^m\], for all numbers \(a\) and \(b\) and any integer \(m\)

Example
Simplify.

a. \((7x^4)^2\)

\[
(7x^4)^2 = 7^2 \cdot (x^4)^2
\]

Power of a Product

\[
= 7^2 \cdot x^{4 \cdot 2}
\]

Power of a Power

\[
= 49 \cdot x^8
\]

Simplify.

b. \((3a^4b^6)^2\)

\[
(3a^4b^6)^2 = 3^2 \cdot (a^4)^2 \cdot (b^6)^2
\]

Power of a Product

\[
= 3^2 \cdot (a^{4 \cdot 2}) \cdot (b^{6 \cdot 2})
\]

Power of a Power

\[
= 9a^8b^{12}
\]

Simplify.

Exercises
Simplify.

1. \((6x^5)^3\)

2. \((5b^{-3})^4\)

3. \((12h^7)^2\)

4. \((-8j^2)^3\)

5. \((11z^{-9})^2\)

6. \((7a^6)^3\)

7. \((4g^{-2})^4\)

8. \((2k^3)^5\)

9. \((6p^7q^6)^3\)

10. \((-9m^9n^4)^3\)

11. \((10f^{-2}g^3)^5\)

12. \((5d^7e^{10})^3\)

13. \((-4s^6t^8)^4\)

14. \((3r^5s^3)^4\)

15. \((8a^2b^3)^3\)

16. \((-10v^{-5}w^3)^4\)
9-6 Skills Practice

Powers of Monomials

Simplify.

1. \((7^2)^3\)  
2. \((12^3)^5\)  
3. \((-15^6)^4\)

4. \((h^3)^{-7}\)  
5. \((6^5)^{10}\)  
6. \((-f^7)^2\)

7. \((k^8)^{-6}\)  
8. \((a^9)^{-3}\)  
9. \((16^5)^8\)

10. \((42^5)^4\)  
11. \((m^3)^{-8}\)  
12. \((p^{-8})^7\)

13. \((90^5)^9\)  
14. \((-3^6)^5\)  
15. \((-b^3)^4\)

16. \((6y^6)^4\)  
17. \((12k^5)^2\)  
18. \((-7c^{-6})^3\)

19. \((11n^5)^3\)  
20. \((3p^{10})^5\)  
21. \((-5g^{-8})^4\)

22. \((4a^2b^3)^5\)  
23. \((-10h^8j^9)^6\)  
24. \((9r^2t^{-3})^2\)

25. \((-8x^{-2}y^{-6})^3\)  
26. \((2g^2h^3)^8\)  
27. \((-3a^4b^3)^4\)

28. \((20t^7u^{12})^3\)  
29. \((40x^5z^{-2})^2\)  
30. \((13f^9g^2)^2\)
9-6 Practice

Powers of Monomials

Simplify.

1. \((19^3)^6\)
2. \((-8^4)^9\)
3. \((28^2)^{-5}\)

4. \((q^8)^{-2}\)
5. \((w^3)^4\)
6. \((-46^{10})^7\)

7. \((b^9)^{-3}\)
8. \((m^5)^{-2}\)
9. \((-103^4)^{12}\)

10. \((88^3)^7\)
11. \((x^{-2})^4\)
12. \((v^{-4})^4\)

13. \((7l^8)^2\)
14. \((-4x^3)^4\)
15. \((-9f^7)^3\)

16. \((12r^{-5})^2\)
17. \((3s^8)^4\)
18. \((-5y^7)^4\)

19. \((10u^5v^{-3})^5\)
20. \((-2h^5i^3)^7\)
21. \((4c^{-6}d^6)^2\)

22. \((9f^5g^{-5})^3\)
23. \((11j^{-4}k^{-2})^2\)
24. \((-3b^5c^{-7})^5\)

25. \((-5x^{-3}y^{-5}z^2)^2\)
26. \((-7a^5b^6c^7)^2\)
27. \((4g^6h^{-2}i^{-8})^6\)

28. \([(4^2)^3]^2\)
29. \((0.6c^{-5})^2\)
30. \((\frac{1}{4}r^4s^8)^3\)

31. GEOMETRY Find the area of a square with sides of length \(6x^2y^5\) units.

32. GEOMETRY Find the volume of a cube with sides of length \(4a^4b^6\) units.

33. ASTRONOMY The diameter of the Sun is \(8.65 \times 10^5\) miles. Use the formula \(A = 3.14 \cdot r^2\) to find the area of the cross section of the Sun at the equator.
1. **AREA** Haley’s math teacher drew the square shown below. Find the area of the square.

![Square Diagram]

\[ d^4 \]

2. **PYRAMIDS** The Great Pyramid in Giza has a square base that measures \(7.55 \times 10^2\) feet. Find the area of the base of the pyramid.

3. **ATOMS** Electrons are found in the nucleus of atoms. The radius of an electron is \(2.8179 \times 10^{-15}\) meters. Use the formula \(V = \frac{4}{3} \cdot 3.14 \cdot r^3\) to find the volume of an electron. Round to the nearest thousandth.

4. **VOLUME** What is the volume of a cube with sides that measure \(5x^2y^3\)? (Hint: \(V = s^3\))

5. **MINERALS** A grain of salt is cubical in shape. Each side measures about \(5 \times 10^{-1}\) mm in length. Use the formula \(SA = 6s^2\) to find the surface area of a grain of salt.

6. **CELESTIAL BODIES** The table below lists the radii of different celestial bodies.

<table>
<thead>
<tr>
<th>Celestial Body</th>
<th>Radius (mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moon</td>
<td>(1.08 \times 10^3)</td>
</tr>
<tr>
<td>Earth</td>
<td>(3.959 \times 10^3)</td>
</tr>
<tr>
<td>Sun</td>
<td>(4.32 \times 10^5)</td>
</tr>
</tbody>
</table>

a. Use the formula \(V = \frac{4}{3} \cdot 3.14 \cdot r^3\) to find the volume of the Moon. Round to the nearest million.

b. Use the formula \(SA = 4 \cdot 3.14 \cdot r^2\) to find the surface area of the Sun. Round to the nearest billion.

c. Use the formula \(A = 3.14 \cdot r^2\) to find the area of the cross section of Earth at the equator. Round to the nearest million.
Enrichment

**Fractional Exponents**

A radical sign (\(\sqrt{\phantom{x}}\)) indicates the square root of a number. The square root is one of the number’s two equal factors. You have seen that both positive and negative integers can be exponents. A fraction can also be an exponent. Fractional exponents can be rewritten in radical form:

\[
\frac{25^{\frac{1}{2}}}{\sqrt{25}} = 5, \text{ since } 5 \cdot 5 = 25
\]

If \(x > 0\) and \(p\) and \(q\) are integers and \(q > 0\), then \(x^{\frac{p}{q}} = \sqrt[q]{x^p}\).

The denominator of the fractional exponent indicates the root:

\[
1,000^{\frac{1}{3}} = \sqrt[3]{1,000} = 10
\]

The cube root of 1,000 is 10 since \(10^3 = 1,000\).

The numerator of the fractional exponent indicates the power to which to raise the root:

\[
10,000^{\frac{3}{2}} = (\sqrt[2]{10,000})^3
\]

\[
= (10)^3 = 1,000
\]

Simplify.

### Example

**Simplify each expression.**

**a.** \(16^{\frac{1}{2}}\)

\[
16^{\frac{1}{2}} = \sqrt{16} = 4
\]

**b.** \(27^{\frac{2}{3}}\)

\[
27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9
\]

The cube root of 27 is 3, since \(3^3 = 27\). Simplify.

### Exercises

**Simplify each expression.**

1. \(81^{\frac{1}{2}}\)
2. \(9^{\frac{1}{2}}\)
3. \(49^{\frac{1}{2}}\)
4. \(36^{\frac{1}{2}}\)

5. \(64^{\frac{1}{2}}\)
6. \(125^{\frac{1}{3}}\)
7. \(216^{\frac{1}{3}}\)
8. \(64^{\frac{1}{3}}\)

9. \(8^{\frac{1}{3}}\)
10. \(512^{\frac{1}{3}}\)
11. \(216^{\frac{2}{3}}\)
12. \(125^{\frac{2}{3}}\)

13. \(64^{\frac{2}{3}}\)
14. \(16^{\frac{3}{4}}\)
15. \(1,000^{\frac{2}{3}}\)
16. \(81^{\frac{3}{4}}\)

---

Chapter 9

41

Glencoe Pre-Algebra
9-7 Study Guide and Intervention

Linear and Nonlinear Functions

Graphs of Nonlinear Functions  Linear functions are relations with a constant rate of change. Graphs of linear functions are straight lines. Nonlinear functions do not have a constant rate of change. Graphs of nonlinear functions are not straight lines.

Example  Determine whether each graph represents a linear or nonlinear function. Explain.

a.  
   ![Graph a]
   This graph is a curve, not a straight line. So, it represents a nonlinear function.

b.  
   ![Graph b]
   This graph is a line. So, it represents a linear function.

Exercises

Determine whether each graph represents a linear or nonlinear function. Explain.

1.  
   ![Graph 1]

2.  
   ![Graph 2]

3.  
   ![Graph 3]

4.  
   ![Graph 4]
9-7 Study Guide and Intervention (continued)

Linear and Nonlinear Functions

Equations and Tables Linear functions have constant rates of change. Their graphs are straight lines and their equations can be written in the form $y = mx + b$. Nonlinear functions do not have constant rates of change and their graphs are not straight lines.

Example 1 Determine whether each equation represents a linear or nonlinear function. Explain.

a. $y = 9$
   This is linear because it can be written as $y = 0x + 9$.

b. $y = x^2 + 4$
   This is nonlinear because the exponent of $x$ is not 1, so the equation cannot be written in the form $y = mx + b$.

Example 2 Determine whether each table represents a linear or nonlinear function. Explain.

a. As $x$ increases by 2, $y$ increases by 8. The rate of change is constant, so this is a linear function.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-7</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>17</td>
</tr>
</tbody>
</table>

b. As $x$ increases by 5, $y$ decreases by a greater amount each time. The rate of change is not constant, so this is a nonlinear function.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>75</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>-125</td>
</tr>
</tbody>
</table>

Exercises

Determine whether each equation or table represents a linear or nonlinear function. Explain.

1. $x + 3y = 9$

2. $y = \frac{8}{x}$

3. $y = 6x(x + 1)$

4. $y = 9 - 5x$

5. | $x$ | $y$ |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>24</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>-6</td>
</tr>
</tbody>
</table>

6. | $x$ | $y$ |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>4</td>
<td>64</td>
</tr>
</tbody>
</table>
Determine whether each graph, equation, or table represents a linear or nonlinear function. Explain.

1. [Graph with a curved line]
2. [Graph with a straight line]
3. [Graph with a parabolic curve]
4. \(y = \frac{x}{2} + 1\)
5. \(y = \frac{2}{x} + 10\)
6. \(y = 8x\)
7. \(y = 6\)
8. \(2x - y = 5\)
9. \(y = x^2 + 4\)
10. \(y + 4x^2 - 1 = 0\)
11. \(2y - 8x + 11 = 0\)
12. \(y = \sqrt{3x} - 2\)
13. | \(x\) | \(y\) |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>-1</td>
</tr>
</tbody>
</table>
14. | \(x\) | \(y\) |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>18</td>
<td>6</td>
</tr>
<tr>
<td>24</td>
<td>10</td>
</tr>
</tbody>
</table>
15. | \(x\) | \(y\) |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>-4</td>
</tr>
<tr>
<td>15</td>
<td>-2</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>
9-7 Practice

Linear and Nonlinear Functions

Determine whether each graph, equation, or table represents a linear or nonlinear function. Explain.

1. [Graph]

4. \(5x - y = 15\)

5. \(3y + 12x^2 = 0\)

6. \(5y - x + 3 = 0\)

7. \(y = 6\sqrt{x} + 10\)

8. \(y = \frac{8}{x}\)

9. \(y = -x^2 + 2\)

10. \[
\begin{array}{c|c}
  x & y \\
  1 & 1.0 \\
  2 & 0.8 \\
  3 & 0.6 \\
  4 & 0.4 \\
\end{array}
\]

11. \[
\begin{array}{c|c}
  x & y \\
  44 & 0 \\
  48 & 2.5 \\
  52 & 5.0 \\
  56 & 7.5 \\
\end{array}
\]

12. \[
\begin{array}{c|c}
  x & y \\
  3 & 1 \\
  6 & -2 \\
  9 & -5 \\
  12 & -14 \\
\end{array}
\]

13. GEOMETRY The graph shows how the area of a square increases as the perimeter increases. Is this relationship linear or nonlinear? Explain.
9-7 Word Problem Practice

Linear and Nonlinear Functions

1. TEMPERATURE In the United States, temperature is most often measured in degrees Fahrenheit. Temperature is measured in degrees Celsius in the metric system. The formula used to convert between these two units of measure is
   \[ F = \frac{9}{5}C + 32 \]
   where \( F \) represents degrees Fahrenheit and \( C \) represents degrees Celsius. Does this equation represent a linear or nonlinear function?

2. COMPUTER GAMES Suppose the function
   \[-0.005d^2 + 0.12d = h\]
   is used to simulate the path of a golf ball that is hit off a tee in a computer game. Does this equation represent a linear or nonlinear function?

3. GASOLINE The table below shows gasoline prices in Springfield during a one-month period. Is the change in gas price a linear function? Explain.

<table>
<thead>
<tr>
<th>Day of the Month</th>
<th>Price per Gallon</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2.57</td>
</tr>
<tr>
<td>4</td>
<td>$2.72</td>
</tr>
<tr>
<td>7</td>
<td>$2.72</td>
</tr>
<tr>
<td>10</td>
<td>$2.88</td>
</tr>
<tr>
<td>13</td>
<td>$2.88</td>
</tr>
<tr>
<td>16</td>
<td>$2.84</td>
</tr>
<tr>
<td>19</td>
<td>$2.76</td>
</tr>
<tr>
<td>21</td>
<td>$2.72</td>
</tr>
<tr>
<td>24</td>
<td>$2.64</td>
</tr>
<tr>
<td>27</td>
<td>$2.60</td>
</tr>
<tr>
<td>30</td>
<td>$2.52</td>
</tr>
</tbody>
</table>

4. FOOTBALL PUNTS The function
   \[ h = -16t^2 + 90t + 1.5 \]
   represents the height \( h \) of the football, in feet, after \( t \) seconds when a punter kicks the ball with an upward velocity of 90 feet per second and his foot meets the ball 1.5 feet off the ground. Is this a linear function of time? Explain.

5. FLIGHT RESEARCH The equation
   \[ h = -16t^2 + 608t + 4482 \]
   represents the height, \( h \), in feet, of a pilot over time, \( t \), in seconds, after he or she has ejected from a jet and falls to Earth with the aid of a parachute. A pilot is flying at an altitude of approximately 10,000 feet and is forced to eject from the jet. The equation \( h = 10000 \) represents an altitude of 10,000 feet.

   a. Which equation is a linear function?

   b. Explain why the other equation is a nonlinear function.
**David R. Hedgley**

African-American mathematician David R. Hedgley, Jr. (1937–) solved one of the most difficult problems in the field of computer graphics—how to program a computer to show any three-dimensional object from a given viewpoint just as the eye would see it. Hedgley’s solution helped researchers in aircraft experimentation. Hedgley received an M.S. in Mathematics from California State University in 1970 and a Ph.D. in Computer Science from Somerset University in England in 1988. Hedgley has received numerous national achievement awards.

Polynomials in three variables are needed to describe some three-dimensional objects. Each variable represents one of the three dimensions: height, width, and depth.

\[ P_1: x^2 + y^2 + z^2 + 10x + 4y + 2z - 19 \]
\[ P_2: 2x^2 + 2y^2 + 2z^2 - 2x - 3y + 5z - 2 \]

1. Add the polynomials \( P_1 \) and \( P_2 \).

2. Subtract the polynomials, \( P_1 \) from \( P_2 \).

If the polynomials above were each set equal to zero, they would form equations describing two different spheres in three-dimensional space, or 3-space. The coordinate plane you studied in Chapter 2 represents two-space. You described most lines in that plane by an equation in two variables. Each point on a line could be written as an ordered pair of numbers \((x, y)\). Each point on any figure in 3-space can be written as an ordered triple of numbers \((x, y, z)\).

3. What are the values of \( x, y, \) and \( z \) for point \( A \) in the diagram?

4. Give the ordered triple representing each of the points \( B \) through \( G \) in the diagram.
Graph Quadratic Functions Functions which can be described by an equation of the form $y = ax^2 + bx + c$, where $a \neq 0$, are called quadratic functions. The graph of a quadratic equation takes the form shown to the right, which is called a parabola.

Just as with linear functions, you can graph quadratic functions by making a table of values.

### Example
Graph $y = x^2 - 3$.

Make a table of values, plot the ordered pairs, and connect the points with a curve.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = x^2 - 3$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>$y = (-2)^2 - 3 = 1$</td>
<td>(-2, 1)</td>
</tr>
<tr>
<td>-1</td>
<td>$y = (-1)^2 - 3 = -2$</td>
<td>(-1, -2)</td>
</tr>
<tr>
<td>0</td>
<td>$y = (0)^2 - 3 = -3$</td>
<td>(0, -3)</td>
</tr>
<tr>
<td>1</td>
<td>$y = (1)^2 - 3 = -2$</td>
<td>(1, -2)</td>
</tr>
<tr>
<td>2</td>
<td>$y = (2)^2 - 3 = 1$</td>
<td>(2, 1)</td>
</tr>
</tbody>
</table>

### Exercises
Graph each function.

1. $y = x^2 + 2$
2. $y = -x^2 + 2$
3. $y = x^2 - 2$
4. $y = 3x^2 - 1$
5. $y = \frac{1}{4}x^2$
6. $y = -2x^2 + 3$
Quadratic Functions

Use Quadratic Functions  Many quadratic functions model real-world situations. You can use graphs of quadratic equations to analyze such situations.

Example  MAPS  The principal of Smithville Elementary wants to paint a map of the U.S. on the cafeteria wall. Before the map can be painted, the rectangular space where the map will go must be painted white. The height of the rectangle will be \( \frac{3}{5} \) the width.

a. Graph the equation that gives the area for the rectangle for different lengths and widths. What is the area of the rectangle with a width of 10 feet? What is the length?

Since area = length \( \times \) width, use the quadratic equation \( y = \frac{3}{5} x^2 \), where \( y \) = the area and \( x \) = the width.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = \frac{3}{5} x^2 )</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>( y = \frac{3}{5} (4)^2 )</td>
<td>(4, 9.6)</td>
</tr>
<tr>
<td>6</td>
<td>( y = \frac{3}{5} (6)^2 )</td>
<td>(6, 21.6)</td>
</tr>
<tr>
<td>8</td>
<td>( y = \frac{3}{5} (8)^2 )</td>
<td>(8, 38.4)</td>
</tr>
<tr>
<td>10</td>
<td>( y = \frac{3}{5} (10)^2 )</td>
<td>(10, 60)</td>
</tr>
<tr>
<td>12</td>
<td>( y = \frac{3}{5} (12)^2 )</td>
<td>(12, 86.4)</td>
</tr>
</tbody>
</table>

The area of the rectangle when the width is 10 feet is 60 square feet. The length is 6 feet.

b. What values of the domain and range are unreasonable? Explain.

Unreasonable values for the domain and range would be any negative numbers because neither the length nor the width can be negative.

Exercise

1. GRAVITY  An object is dropped from a height of 300 feet. The equation that gives the object’s height in feet \( h \) as a function of time \( t \) is \( h = -16t^2 + 300 \). Graph this equation and interpret your graph. What was the height of the object after 4 seconds?
Skills Practice

Quadratic Functions

Graph each function.

1. \( y = 5x^2 \)

2. \( y = -x^2 \)

3. \( y = -5x^2 \)

4. \( y = x^2 - 1 \)

5. \( y = x^2 + 4 \)

6. \( y = -2x^2 + 2 \)

7. \( y = x^2 - 4 \)

8. \( y = 2x^2 - 2 \)
Graph each function.

1. \( y = 0.4x^2 \)

2. \( y = -\frac{1}{2}x^2 \)

3. \( y = -2x^2 - 1 \)

4. \( y = 3x^2 - 4 \)

5. **WINDOWS** A window maker has 25 feet of wire to frame a window. One side of the window is \( x \) feet and the other side is \( 9 - x \) feet.

   a. Write an equation to represent the area \( A \) of the window.

   b. Graph the equation you wrote in part a.

   c. If the area of the window is 18 square feet, what are the two possible values of \( x \)?
1. **RACING** Between the ages of 8 and 16, Houston native Erica Enders won 37 junior dragster races. The distance her car travels down the drag strip can be expressed by the equation \( d = \frac{1}{2}at^2 \), where \( a \) is the rate of acceleration and \( t \) is time. Suppose her car accelerates at a rate of 49.5 feet per second. Find the number of feet her car traveled after 7 seconds.

2. **PHYSICS** The top of the Leaning Tower of Pisa is 185 feet above the ground. Suppose an object is dropped from the top of the Leaning Tower of Pisa. The height \( h \) in feet of the object, after \( t \) seconds, is represented by the equation \( h = 185 - 16t^2 \). How far from the ground is it after 3 seconds?

3. **VISTAS** The Texas State Capitol building is 311 feet tall. The formula \( a = \frac{2}{3}d^2 \) represents the number of miles \( d \) that a person can see from an altitude of \( a \) feet. Graph the function and use it to estimate how far you could see from the top of the Texas State Capitol.

4. **FIREWORKS** The largest annual pyrotechnic display in North America is *Thunder over Louisville* held to kick off the Kentucky Derby Festival. The table shows the larger shell sizes and their corresponding velocities.

<table>
<thead>
<tr>
<th>Shell Size (in.)</th>
<th>Initial Velocity (ft/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>235</td>
</tr>
<tr>
<td>10</td>
<td>263</td>
</tr>
<tr>
<td>12</td>
<td>287.5</td>
</tr>
<tr>
<td>24</td>
<td>393</td>
</tr>
<tr>
<td>36</td>
<td>481</td>
</tr>
</tbody>
</table>

*Source: ThinkQuest*

a. The equation \( h = -16t^2 + 235t + 3 \) represents the height \( h \) in feet of an 8-inch shell \( t \) seconds after it is launched from 3 feet with an initial velocity of 235 feet per second. Graph the equation.

b. How high is the shell after 5 seconds?
Translating Quadratic Graphs

When a figure is moved to a new position without undergoing any rotation, then the figure is said to have been translated to the new position.

The graph of a quadratic equation in the form \( y = (x - b)^2 + c \) is a translation of the graph of \( y = x^2 \).

Start with a graph of \( y = x^2 \).

Slide to the right 4 units.

\[ y = (x - 4)^2 \]

Then slide up 3 units.

\[ y = (x - 4)^2 + 3 \]

The following equations are in the form \( y = x^2 + c \). Graph each equation.

1. \( y = x^2 + 1 \)  
2. \( y = x^2 + 2 \)  
3. \( y = x^2 - 2 \)

The following equations are in the form \( y = (x + b)^2 \). Graph each equation.

4. \( y = (x - 1)^2 \)  
5. \( y = (x - 3)^2 \)  
6. \( y = (x + 2)^2 \)
A *family of graphs* is a group of graphs that have at least one characteristic in common. You can use a spreadsheet to study the characteristics of families of quadratic graphs.

**Graph the quadratics**

\[ y = x^2, \ y = 2x^2, \text{ and } y = 4x^2. \] What are the similarities and differences among the graphs?

**Step 1** Use Column A for the values of \( x \) and Columns B, C, and D for the values of \( y \). Exponents are entered using the ^ symbol. For example, cell B2 contains the formula A2^2.

**Step 2** To create a graph from the data, select the data in Columns A, B, C, and D and choose Chart from the Insert menu. Select an XY (Scatter) chart with a smooth line to show the graphs.

The graphs of all three functions pass through the point at (0, 0).

The graph of \( y = x^2 \) is wider than the graph of \( y = 2x^2 \). The graph of \( y = 2x^2 \) is wider than the graph of \( y = 4x^2 \).

**Exercises**

1. Make a conjecture about the graph of \( y = \frac{1}{2}x^2 \) as compared to the graphs above. Use the spreadsheet to graph \( y = \frac{1}{2}x^2 \) and verify your conjecture.

2. Graph the quadratics \( y = x^2, \ y = x^2 + 2, \text{ and } y = x^2 - 3. \) What are the similarities and differences among the graphs?
Cubic Functions  Functions which can be described by an equation of the form \( y = ax^3 + bx^2 + cx + d \), where \( a \neq 0 \), are called cubic functions. The graph of a cubic equation takes the form shown to the right.

Just as with linear and quadratic functions, you can graph cubic functions by making a table of values.

### Example

Graph \( y = 2x^3 - 1 \).

Make a table of values, plot the ordered pairs, and connect the points with a curve.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = 2x^3 - 1 )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>( y = 2(-1)^3 - 1 = -3 )</td>
<td>(-1, -3)</td>
</tr>
<tr>
<td>0</td>
<td>( y = 2(0)^3 - 1 = -1 )</td>
<td>(0, -1)</td>
</tr>
<tr>
<td>1</td>
<td>( y = 2(1)^3 - 1 = 1 )</td>
<td>(1, 1)</td>
</tr>
<tr>
<td>1.2</td>
<td>( y = 2(1.2)^3 - 1 \approx 2.5 )</td>
<td>(1.2, 2.5)</td>
</tr>
</tbody>
</table>

### Exercises

Graph each function.

1. \( y = x^3 + 2 \)
2. \( y = -x^3 + 2 \)
3. \( y = x^3 - 2 \)
4. \( y = 2x^3 \)
5. \( y = -2x^3 + 2 \)
6. \( y = \frac{5}{6}x^3 - 1 \)
Cubic and Exponential Functions

Exponential Functions  In linear, quadratic, and cubic functions, the variable is the base. Exponential functions are functions in which the variable is the exponent rather than the base. An exponential function is a function that can be described by an equation of the form \( y = a^x + c \), where \( a \neq 0 \) and \( a \neq 1 \).

Example  Graph \( y = 3^x - 6 \).

First, make a table of ordered pairs. Then graph the ordered pairs.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = 3^x - 6 )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>( y = 3^{-2} - 6 \approx -5.9 )</td>
<td>(-2, -5.9)</td>
</tr>
<tr>
<td>-1</td>
<td>( y = 3^{-1} - 6 \approx -5.7 )</td>
<td>(-1, -5.7)</td>
</tr>
<tr>
<td>0</td>
<td>( y = 3^0 - 6 = -5 )</td>
<td>(0, -5)</td>
</tr>
<tr>
<td>1</td>
<td>( y = 3^1 - 6 = -3 )</td>
<td>(1, -3)</td>
</tr>
<tr>
<td>2</td>
<td>( y = 3^2 - 6 )</td>
<td>(2, 3)</td>
</tr>
</tbody>
</table>

Exercises

Graph each function.

1. \( y = 2^x \)

2. \( y = 3^x + 2 \)

3. \( y = 2^x - 1 \)

4. \( y = 2^x + 1 \)

5. \( y = 3^x - 3 \)

6. \( y = 4^x - 6 \)
Graph each function.

1. \( y = 5x^3 \)
   ![Graph of \( y = 5x^3 \)]

2. \( y = -5x^3 \)
   ![Graph of \( y = -5x^3 \)]

3. \( y = x^3 + 4 \)
   ![Graph of \( y = x^3 + 4 \)]

4. \( y = x^3 - 4 \)
   ![Graph of \( y = x^3 - 4 \)]

5. \( y = 2x^3 + 3 \)
   ![Graph of \( y = 2x^3 + 3 \)]

6. \( y = -x^3 \)
   ![Graph of \( y = -x^3 \)]

7. \( y = 2x - 4 \)
   ![Graph of \( y = 2x - 4 \)]

8. \( y = 3x - 7 \)
   ![Graph of \( y = 3x - 7 \)]
Graph each function.

1. \( y = 0.4x^3 \)
2. \( y = -2x^3 - 1 \)
3. \( y = x^3 + 0.5 \)
4. \( y = \frac{1}{5}x^3 \)
5. \( y = 3^x + 0.75 \)
6. \( y = 3^x - 4 \)

7. **E-MAIL** Mike forwarded an e-mail to 5 friends. Each of those 5 friends forwarded it to 5 of their friends. Each of those friends forwarded it to five friends and so on. The function \( N = 5^x \) represents the total number of e-mails forwarded, where \( x \) is the stage of the e-mails. Graph the function. In what stage will the number of e-mails forwarded be at least 625?
1. VOLUME The equation \( y = \frac{4}{3} \pi x^3 \) represents the volume of a sphere. Graph the equation in the first quadrant.

2. GEOMETRY Write the function for the volume of a cone as a function of a radius \( r \) units if the height equals the radius. Then graph the function.

3. MONEY Sam’s teacher wrote the problem shown below on the board.

   You have a choice of receiving 1 million dollars all at once, or, to receive a penny one day, then double this number of pennies the next day, then double that number on the third day and so on, for 30 days. Which is the better deal?

   The function representing the number of pennies is \( y = 2^{x-1} \), where \( x \) is the number of days. Graph the function. Is it better to take the million dollars or the pennies? Explain your answer.

4. MARATHONS Helen runs 5 miles every other day. She wants to increase the number of miles she runs to train for a marathon. She plans to increase the mileage according to the function \( y = 5(1.1)^x \), where \( x \) represents the number of times she goes running. Graph the function. On which day will Helen first run 26 miles?

5. POPULATION GROWTH The population of the town of Carlyll is growing at rate of 5.5% per year. There are currently 1,050 people in the town.

   a. Graph the function \( y = 1,050(1.055)^x \), where \( x \) represents the number of years.

   b. Identify the y-intercept and explain what it means.

   c. Estimate the number of people who will live in Carlyll in 12 years.

   d. Estimate the number of people who lived in Carlyll 5 years ago.
**9-9 Enrichment**

**Writing Exponential Growth and Decay Equations**

Exponential functions can be used to represent real-world situations involving both exponential growth and exponential decay. To determine whether an equation in the form $y = a \cdot b^x$ represents exponential growth or decay, look at the value of $b$. If the value of $b$ is greater than 1, the equation represents exponential growth. If the value of $b$ is less than 1, the equation represents exponential decay.

**Example**

<table>
<thead>
<tr>
<th>Words: The population of a town with 1,000 residents is increasing at a rate of 8% per year.</th>
<th>Words: The population of a town with 1,000 residents is decreasing at a rate of 8% per year.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Symbols:</strong> $y = 1000(1.08)^x$</td>
<td><strong>Symbols:</strong> $y = 1000(0.92)^x$</td>
</tr>
<tr>
<td><strong>Graph:</strong> This shows exponential growth.</td>
<td><strong>Graph:</strong> This shows exponential decay.</td>
</tr>
</tbody>
</table>

A simple formula for writing exponential growth and decay equations is shown below.

<table>
<thead>
<tr>
<th>Growth</th>
<th>Decay</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = a(1 + r)^x$</td>
<td>$y = a(1 - r)^x$</td>
</tr>
</tbody>
</table>

Where:
- $a =$ initial amount before growth/decay
- $r =$ rate of growth/decay
- $x =$ number of time intervals

**Exercises**

Write an equation to represent each situation.

1. Simple interest on $600 at 3%.
2. There are 200 competitors in a tournament. One-quarter of the competitors are eliminated at the end of each round.
3. The median home price of $350,000 increases at a rate of 4% a year.
4. A cable company had 10,000 subscribers. The number of subscribers decreased by 2.2% each month.
5. A barometric pressure reading of 30 decreases by about 12% per kilometer above sea level.
Multiple Choice

Read each question. Then fill in the correct answer.

1. ◯ ◯ ◯ ◯ 5. ◯ ◯ ◯ ◯
2. ◯ ◯ ◯ ◯ 6. ◯ ◯ ◯ ◯
3. ◯ ◯ ◯ ◯ 7. ◯ ◯ ◯ ◯
4. ◯ ◯ ◯ ◯ 8. ◯ ◯ ◯ ◯

Short Response/Gridded Response

Record your answer in the blank.

For gridded response questions, also enter your answer in the grid by writing each number or symbol in a box. Then fill in the corresponding circle for that number or symbol.

9. ______________
10. ______________ (grid in)
11. a. ______________
   b. ______________
   c. ______________
12. ______________ (grid in)
13. ______________

Extended Response

Record your answers for Question 14 on the back of this paper.
Rubric for Scoring Extended Response

(Use to score the Extended Response question on page 531 of the Student Edition.)

General Scoring Guidelines

- If a student gives only a correct numerical answer to a problem but does not show how he or she arrived at the answer, the student will be awarded only 1 credit. All extended response questions require the student to show work.

- A fully correct answer for a multiple-part question requires correct responses for all parts of the question. For example, if a question has three parts, the correct response to one or two parts of the question that required work to be shown is not considered a fully correct response.

- Students who use trial and error to solve a problem must show their method. Merely showing that the answer checks or is correct is not considered a complete response for full credit.

Exercise 14 Rubric

<table>
<thead>
<tr>
<th>Score</th>
<th>Specific Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>For part a, it can be determined from the graph that the rocket reached its peak height at 4 seconds. Replacing x with 4 in the equation for part b gives ( y = -16(4)^2 + 128(4); \ y = 256 ). So, the rocket reached a peak height of 256 feet. For part c, it can be determined from the graph that the rocket landed after 8 seconds. This can be checked by replacing x with 8 in the equation: ( y = -16(8)^2 + 128(8); \ y = 0 ).</td>
</tr>
<tr>
<td>3</td>
<td>A generally correct solution, but may contain minor flaws in reasoning or computation.</td>
</tr>
<tr>
<td>2</td>
<td>A partially correct interpretation and/or solution to the problem.</td>
</tr>
<tr>
<td>1</td>
<td>A correct solution with no evidence or explanation.</td>
</tr>
<tr>
<td>0</td>
<td>An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given.</td>
</tr>
</tbody>
</table>
Chapter 9 Quiz 1
(Lessons 9-1 through 9-3)

Write each expression using exponents.
1. \(x \cdot x \cdot x\)  
2. \((9 \cdot 9 \cdot 9) \cdot (9 \cdot 9)\)

Evaluate each expression if \(x = 2\), \(a = 3\), and \(b = 2\).
3. \(12x^4\)  
4. \(a^2b^3\)

Write the prime factorization of each number. Use exponents for repeated factors.
5. 27  
6. 63

Factor each monomial completely.
7. \(21b\)  
8. \(30x^2y\)

Find each product or quotient.
9. \(-r^3(3r^6)\)  
10. \(\frac{a^{16}}{a^4}\)

Chapter 9 Quiz 2
(Lessons 9-4 and 9-5)

Write each expression using a positive exponent.
1. \(7^{-5}\)  
2. \(y^{-3}\)

Evaluate each expression if \(x = 3\) and \(y = 2\).
3. \(x^{-4}\)  
4. \((xy)^{-3}\)  
5. \(y^{-2}\)

Write each number in scientific notation.
6. 0.0000001602
7. 200,000,000
8. 15

Choose the greater number in each pair.
9. \(4.62 \times 10^{-3}\), 0.000462  
10. \(6.25 \times 10^4\), 6500
Chapter 9 Quiz 3

(Lessons 9-6 and 9-7)

Simplify.
1. \((2x^4)^3\)  
2. \((-3y^2)^{-3}\)

For Questions 3 and 4, determine whether each graph or table represents a linear or nonlinear function. Explain.

3. 

![Graph 3](image)

4. 

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>40</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
</tr>
<tr>
<td>12</td>
<td>20</td>
</tr>
<tr>
<td>19</td>
<td>10</td>
</tr>
</tbody>
</table>

5. MULTIPLE CHOICE Which equation describes a linear function?
   A \(a = (a + 3)5a\)  
   B \(x + 9y = 15\)  
   C \(-4t^2 + 3r = 7\)  
   D \(a = 3m^3 + 1\)

Chapter 9 Quiz 4

(Lessons 9-8 and 9-9)

For Questions 1 and 2, graph each function.
1. \(y = -x^3\)

2. \(y = 2^x - 1\)

3. Graph \(y = 2x^2\) and \(y = x^2\) on the same coordinate plane. Describe their similarities and differences.
Part I  Write the letter for the correct answer in the blank at the right of each question.

1. Which shows the expression \(a \cdot a \cdot a \cdot (b + 3) \cdot (b + 3)\) using exponents?
   \[A \quad a^3 b^2 \quad B \quad 3a^3 (b + 3) \quad C \quad a^3 (b + 3)^2 \quad D \quad a^2 (b + 3)^3\]
   1. ____

2. Evaluate \(2x^2\) if \(x = 3\).
   \[F \quad 36 \quad G \quad 18 \quad H \quad 12 \quad J \quad 8\]
   2. ____

3. Write the prime factorization of 84.
   \[A \quad 2 \cdot 2 \cdot 21 \quad B \quad 2 \cdot 2 \cdot 2 \cdot 3 \cdot 7 \quad C \quad 4 \cdot 21 \quad D \quad 2 \cdot 2 \cdot 3 \cdot 7\]
   3. ____

4. Choose the number that is prime.
   \[F \quad 21 \quad G \quad 37 \quad H \quad 49 \quad J \quad 55\]
   4. ____

5. Find the product of \(g^6 (8g^2)\).
   \[A \quad 8g^{12} \quad B \quad 8g^8 \quad C \quad 64g^{12} \quad D \quad 64g^8\]
   5. ____

6. Which has the same value as \(p^{-3}\)?
   \[F \quad -3p \quad G \quad -p^3 \quad H \quad \frac{1}{p^3} \quad J \quad -\frac{1}{p^3}\]
   6. ____

Part II


8. Write the prime factorization of 36.

Factor each number or monomial completely.

9. 66
   9. ____________

10. \(24x^2y^2\)
    10. ____________

Find each product or quotient.

11. \((-2d^5)(7d^9)\)
    11. ____________

12. \(\frac{n^{10}}{n^6}\)
    12. ____________

13. Write \(\frac{1}{49}\) as an expression using a negative exponent other than \(-1\).
    13. ____________
Chapter 9 Vocabulary Test

Write the letter of the term that best matches each statement or phrase.

1. written in the form \( y = ax^2 + c \), where \( a \neq 0 \) and \( a \neq 1 \)
   - a. power
   - b. parabola
   - c. exponent
   - d. quadratic function
   - e. monomial
   - f. cubic function
   - g. exponential function
   - h. composite number
   - i. prime number

2. a whole number with exactly two factors, 1 and itself
   - The letter that best matches this statement is **c. exponent**.

3. tells how many times a number is used as a factor
   - The letter that best matches this statement is **e. monomial**.

4. the shape of the graph of the equation \( y = x^2 + 1 \)
   - The letter that best matches this statement is **b. parabola**.

5. written in the form \( y = ax^2 + bx + c \), where \( a \neq 0 \)
   - The letter that best matches this statement is **d. quadratic function**.

6. written in the form \( y = ax^3 + bx + cx + d \), where \( a \neq 0 \)
   - The letter that best matches this statement is **f. cubic function**.

7. a number, variable, or product of numbers and/or variables
   - The letter that best matches this statement is **i. prime number**.

8. the number 51
   - The letter that best matches this statement is **h. composite number**.

9. a number that is expressed using an exponent
   - The letter that best matches this statement is **a. power**.

Define each term in your own words.

10. scientific notation

   - The term scientific notation refers to a method of expressing numbers in a standard form, typically in the form \( a \times 10^n \), where \( 1 \leq a < 10 \) and \( n \) is an integer.

11. base

   - In the context of algebra, the base is the number or variable that is raised to a power. For example, in \( y = ax^2 + bx + c \), \( a \) is the base.

12. nonlinear function

   - A nonlinear function is a function that cannot be represented by a straight line on a standard graph. Examples include quadratic and cubic functions, which have graphs that are curves rather than straight lines.
Write the letter for the correct answer in the blank at the right of each question.

1. Write \((6)(6)(6)\) using exponents.
   - A \(6^6\)
   - B \(3^6\)
   - C \(6^3\)
   - D \(1^3\)

2. Evaluate \(k^3\) if \(k = 2\).
   - F 2
   - G 6
   - H 8
   - J 27

3. Write the prime factorization of 18.
   - A \(2 \cdot 9\)
   - B \(2 \cdot 3 \cdot 3\)
   - C \(2 \cdot 2 \cdot 3\)
   - D \(3 \cdot 6\)

4. Factor 72 completely.
   - F \(3^2 \cdot 2^2\)
   - G \(3^3 \cdot 2^3\)
   - H \(3 \cdot 2^3\)
   - J \(3^2 \cdot 2^3\)

5. Factor 35\(x^2y\) completely.
   - A \(5 \cdot 7 \cdot x \cdot x \cdot y\)
   - B \(35 \cdot 1 \cdot x \cdot x \cdot y\)
   - C \(5 \cdot 7 \cdot x \cdot y \cdot y\)
   - D \(5 \cdot 7 \cdot x^2 \cdot y\)

For Questions 6–8, find each product or quotient.

6. \(4^5 \cdot 4^2\)
   - F \(4^{52}\)
   - G \(16^7\)
   - H \(4^{10}\)
   - J \(4^7\)

7. \(\frac{t^3}{t}\)
   - A \(1^3\)
   - B \(t^3\)
   - C \(t^4\)
   - D \(t^2\)

8. \(m^4 \cdot m\)
   - F \(4^m\)
   - G \(m^5\)
   - H \(m^4\)
   - J \(2m^4\)

9. Write \(\frac{1}{3^5}\) using a negative exponent.
   - A \(5^{-3}\)
   - B \(3^{-5}\)
   - C \(-3^5\)
   - D \(-5^3\)

10. Evaluate \(y^{-2}\) if \(y = 4\).
    - F \(-\frac{1}{8}\)
    - G \(-\frac{1}{16}\)
    - H \(\frac{1}{16}\)
    - J 8

11. Express \(2.36 \times 10^{-2}\) in standard form.
    - A 236
    - B 0.236
    - C 0.0236
    - D \(-236\)
12. The speed of light is 300,000,000 meters per second. Express this number in scientific notation.

F  $3.0 \times 10^8$  G  $30.0 \times 10^8$  H  $300 \times 10^3$  J  $0.03 \times 10^7$  12. ____

13. Choose the true statement.

A  $3.1 \times 10^5 < 2.7 \times 10^5$  C  $5.4 \times 10^4 > 3.7 \times 10^7$

B  $1.8 \times 10^{-1} > 1.1 \times 10^1$  D  $3.7 \times 10^{-4} < 3.4 \times 10^{-1}$  13. ____

For Questions 14 and 15, simplify each expression.

14. $(c^7)^3$

F  $3c^7$  G  $c^{21}$  H  $c^{10}$  J  $c^{21}$  14. ____

15. $(6m^6)^2$

A  $36m^{12}$  B  $36m^8$  C  $12m^{12}$  D  $12m^6$  15. ____

16. Which equation describes a nonlinear function?

F  $x = \frac{y}{4}$  G  $x = 3x + 7$  H  $y = x^2y$  J  $y = 5(x - 1)$  16. ____

17. Which type of function does the graph shown at the right represent?

A  linear  C  cubic  

B  nonlinear  D  quadratic  17. ____

18. Choose the graph that represents $y = x^3 + 1$.

F  

G  

H  

J  18. ____

19. Choose the equation that represents the graph at the right.

A  $y = -x^2$  C  $y = x^3$

B  $y = x^2$  D  $y = -x^3$  19. ____

20. Which describes the equation $y = 3^x + 6$?

F  linear  G  quadratic  H  cubic  J  exponential  20. ____

Bonus  Dyenitha is renting tables for her wedding reception. She can choose from tables that seat 6 or 8. If she is expecting 176 people and wants the same number of people at each table, which size table should she order?  B: ___________
Write the letter for the correct answer in the blank at the right of each question.

   - A \(5 + y\)
   - B \(5y\)
   - C \(5^7\)
   - D \(y^5\)
   1. ____

2. Evaluate \(3m^2\) if \(m = 5\).
   - F \(21\)
   - G \(30\)
   - H \(75\)
   - J \(225\)
   2. ____

3. Write the prime factorization of 36.
   - A \(2 \cdot 2 \cdot 9\)
   - B \(2 \cdot 2 \cdot 2 \cdot 2\)
   - C \(6 \cdot 6\)
   - D \(2 \cdot 2 \cdot 3 \cdot 3\)
   3. ____

4. Factor 98 completely.
   - F \(2 \cdot 7\)
   - G \(2 \cdot 7^2\)
   - H \(7 \cdot 2^2\)
   - J \(7^2 \cdot 2^2\)
   4. ____

5. Factor \(20x^2y\) completely.
   - A \(2 \cdot 2 \cdot 5 \cdot x \cdot x \cdot y\)
   - C \(4 \cdot 5 \cdot x \cdot x \cdot y\)
   - B \(2 \cdot 25x^2y\)
   - D \(2 \cdot 10 \cdot x \cdot x \cdot y\)
   5. ____

For Questions 6–8, find each product or quotient.

6. \(8^4 \cdot 8^4\)
   - F \(64^{16}\)
   - G \(64^8\)
   - H \(8^{16}\)
   - J \(8^8\)
   6. ____

7. \(\frac{4^6}{4^2}\)
   - A \(1^4\)
   - B \(4^3\)
   - C \(4^4\)
   - D \(4^8\)
   7. ____

8. \(\frac{m^5}{m}\)
   - F \(m^4\)
   - G \(5\)
   - H \(m^6\)
   - J \(1^5\)
   8. ____

9. Write \(\frac{1}{5^6}\) as an expression using a negative exponent.
   - A \(6^{-5}\)
   - B \(5^{-6}\)
   - C \(-5^{-6}\)
   - D \(-6\)
   9. ____

10. Evaluate \(b^{-3}\) if \(b = 3\).
    - F \(-9\)
    - G \(-\frac{1}{27}\)
    - H \(\frac{1}{27}\)
    - J \(27\)
    10. ____

11. Express \(6.17 \times 10^5\) in standard form.
    - A \(61,700,000\)
    - B \(617,000\)
    - C \(0.0000617\)
    - D \(0.00000617\)
    11. ____
12. A red blood cell is about $7.5 \times 10^{-4}$ centimeter long. Express this number in standard form.

F 7500   G 0.07500   H 0.0075   J 0.00075  12._____

13. Choose the number that is greater than $2.7 \times 10^4$.

A 26,000   B $1.4 \times 10^5$   C $3.1 \times 10^{-6}$   D $2.5 \times 10^4$  13._____

For Questions 14 and 15, simplify each expression.

14. $(k^4)^{11}$

F $k^{44}$   G $k^{15}$   H $k^{411}$   J $11k^4$  14._____

15. $(2p^3)^{-4}$

A $-2p^{20}$   B $-16p^{20}$   C $\frac{1}{2p^{20}}$   D $\frac{1}{16p^{20}}$  15._____

16. Which equation describes a nonlinear function?

F $y = 1.3x$   G $y = \frac{4x}{7}$   H $y = x^3 - 5$   J $12 = 3x + 4y$  16._____

17. The graph shown at the right represents a function that is

A linear   C cubic
B nonlinear   D quadratic  17._____

18. Choose the graph that represents $y = 3x^3 + 1$.

F   G   H   J  18._____  

19. Choose the equation that represents the graph shown at the right.

A $x^3 - 3$   C $x^2 + 3$
B $-x^3 - 3$   D $-x^2 + 3$  19._____

20. Which describes the equation $y = -2^x - 1$?

F cubic   G quadratic   H exponential   J linear  20._____

Bonus Order the planets in the table at the right from least to greatest diameter.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Diameter (mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uranus</td>
<td>$3.18 \times 10^4$</td>
</tr>
<tr>
<td>Neptune</td>
<td>$3.08 \times 10^4$</td>
</tr>
<tr>
<td>Earth</td>
<td>$7.93 \times 10^3$</td>
</tr>
</tbody>
</table>

B: __________
Write the letter for the correct answer in the blank at the right of each question.

1. Write \((m)(m)(m)(m)\) using exponents.
   A 4 + m     B 4m     C 4\^m    D \(m^4\)  1. ___

2. Evaluate \(3m^3\) if \(m = 6\).
   F 54     G 216     H 648     J 5832  2. ___

3. Write the prime factorization of 24.
   A 2 \cdot 2 \cdot 3     B 2 \cdot 2 \cdot 2 \cdot 3     C 4 \cdot 6     D 2 \cdot 2 \cdot 3 \cdot 3  3. ___

4. Factor 50 completely.
   F 2 \cdot 5     G 1 \cdot 5^2     H 5 \cdot 2^2     J 5^2 \cdot 2  4. ___

5. Factor 28xy\(^2\) completely.
   A 4 \cdot 7 \cdot x \cdot y \cdot y     C 1 \cdot 28 \cdot x \cdot y \cdot y
   B 2 \cdot 2 \cdot 7 \cdot x \cdot y \cdot y     D 2 \cdot 7 \cdot x \cdot y^2  5. ___

For Questions 6–8, find each product or quotient.

6. \(10^3 \cdot 10^6\)
   F \(10^{36}\)     G \(100^3\)     H \(10^{18}\)     J \(10^9\)  6. ___

7. \((3y^2)(6y^3)\)
   A \(18y^5\)     B \(9y^4\)     C \(18y^6\)     D \(9y^5\)  7. ___

8. \(x^6 \div x\)
   F \(1^6\)     G 6     H \(x^7\)     J \(x^5\)  8. ___

9. Write \(\frac{1}{4^7}\) as an expression using a negative exponent.
   A \(4^{-7}\)     B \(-4^{-7}\)     C \(-7\)     D \(-7^4\)  9. ___

10. Evaluate \(t^{-2}\) if \(t = 3\).
    F \(-9\)     G \(-\frac{3}{2}\)     H \(-\frac{1}{9}\)     J \(\frac{1}{9}\)  10. ___

11. Express \(1.08 \times 10^{-3}\) in standard form
    A \(-1.08\)     B \(-0.108\)     C \(0.00108\)     D \(0.108\)  11. ___
12. Bacteria are among the smallest living things. Some of the largest bacteria measure $7.87 \times 10^{-5}$ inch across. Express this number in standard form.

- F $0.0000787$
- G $0.000787$
- H $0.00787$
- J $787,000$

13. Choose the number that is less than $3.4 \times 10^{-4}$.

- A $2.1 \times 10^6$
- B $43,000$
- C $2.1 \times 10^2$
- D $5.4 \times 10^{-6}$

For Questions 14 and 15, simplify each expression.

14. $(d^8)^3$

- F $d^{24}$
- G $d^{11}$
- H $d^{83}$
- J $3d^8$

15. $(3v^4)^3$

- A $3v^{12}$
- B $3v^7$
- C $27v^{12}$
- D $27v^7$

16. Which equation represents a nonlinear function?

- F $y = x^3 + 1$
- G $y = \frac{3x}{8}$
- H $15 = 2x + 3y$
- J $1.7x = y$

17. The graph shown at the right represents a function that is

- A linear
- B nonlinear
- C cubic
- D quadratic

18. Choose the graph that represents $y = x^2 - 1$.

- F
- G
- H
- J

19. Choose the equation that represents the graph shown at the right.

- A $y = -x^2 - 2$
- B $y = x^2 + 2$
- C $y = -x^3 + 2$
- D $y = x^3 - 2$

20. Which describes the equation $y = 4^x + 3$?

- F cubic
- G exponential
- H quadratic
- J linear

Bonus: Order the planets in the table at the right from least to greatest diameter.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Diameter (mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturn</td>
<td>$1.21 \times 10^6$</td>
</tr>
<tr>
<td>Uranus</td>
<td>$5.11 \times 10^4$</td>
</tr>
<tr>
<td>Jupiter</td>
<td>$1.43 \times 10^5$</td>
</tr>
</tbody>
</table>

2. Evaluate \(7a^3\) if \(a = 2\).

Write the prime factorization of each number or monomial.

3. 88

4. \(42a^3x\)

Find each product. Express using exponents.

5. \(m^4 \cdot m^3\)

6. \((11x^3y)(5y)\)

Find each quotient. Express using exponents.

7. \(\frac{(-4)^7}{(-4)^2}\)

8. \(\frac{b^{11}}{b}\)

9. Write \(\frac{1}{3^9}\) using a negative exponent other than \(-1\).

10. Evaluate \(b^{-3}\) if \(b = 5\).

11. Express \(5.09 \times 10^{-4}\) in standard form.

12. Scientists have discovered that many mammals can expect to live for 1.5 billion heartbeats. Write this number in scientific notation.

13. The distance between Saturn and Mars is \(7.53 \times 10^8\) miles. The distance between Mars and Mercury is \(8.37 \times 10^7\) miles. Is Mars closer to Saturn or to Mercury?
For Questions 14 and 15, simplify each expression.

14. \((d^8)^3\)  

15. \((3v^4)^3\)  

Determine whether each graph or table represents a linear or nonlinear function. Explain.

16.  

17.  

For Questions 18–20, graph each function.

18. \(y = 3x^2\)  

19. \(y = x^2 + 2\)  

20. \(y = 2^x - 3\)  

Bonus  Write the prime factorization of 1188. Use exponents for repeated factors.

B: ________________
Chapter 9 Test, Form 2D


2. Evaluate $5y^4$ if $y = 3$.

Write the prime factorization of each number or monomial.

3. 68

4. $32p^2x$

Find each product. Express using exponents.

5. $d^3 \cdot d^2$

6. $(12x^2y)(3y)$

Find each quotient. Express using exponents.

7. $\frac{(-3)^5}{(-3)^2}$

8. $\frac{a^{16}}{a}$

9. Write 0.001 using a negative exponent other than $-1$.

10. Evaluate $a^{-4}$ if $a = 2$.

11. Express $4.68 \times 10^{-4}$ in standard form.

12. The number of different hands possible in the game of bridge is about 635 billion. Write this number in scientific notation.

13. The number of neurons in the neocortex of the human brain is $3.0 \times 10^{10}$. The neocortex of a gorilla contains $7.5 \times 10^8$ neurons. Which mammal has more neurons?
For Questions 14 and 15, simplify each expression.

14. \((z^12)^4\)

15. \((2u^5)^5\)

Determine whether each graph or table represents a \textit{linear} or \textit{nonlinear} function. Explain.

16. 

17. 

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

For Questions 18–20, graph each function.

18. \(y = 2x^2\)

19. \(y = x^3 - 1\)

20. \(y = 3^x\)

B: Write the prime factorization of 1584. Use exponents for repeated factors.
1. Write \(3 \cdot 3 \cdot x \cdot x \cdot y\) using exponents.

2. Evaluate \(4a^4 + 3b\) if \(a = -2\) and \(b = 3\).

Determine whether each number is prime or composite.

3. 211

4. 57

For Questions 5–8, find each product or quotient. Express using exponents.

5. \((-5x^3)(3x^2)\)

6. \((4r^4t)(rt^2)\)

7. \(\frac{a^3b}{b^5a^2}\)

8. \(\left(\frac{m^4}{3m}\right)\left(\frac{12}{m^2}\right)\)

9. Write \(\frac{1}{36}\) using a negative exponent other than \(-1\).

10. Evaluate \((bc)^{-5}\) if \(b = -3\) and \(c = -1\).

11. Express \(1.057 \times 10^{-4}\) in standard form.

12. To find how many seconds it takes light to travel from the Sun to Earth, divide the total distance, 93,000,000 miles, by the distance light travels in one second, 186,000 miles. Write the result in scientific notation.

13. Order \(3.13 \times 10^{-4}, 0.0313, 3.03 \times 10^{-4}, 0.00303, \) and \(3.0 \times 10^{-4}\) from least to greatest.
For Questions 14 and 15, simplify each expression.

14. \((n^5)^{40}\)

15. \((4a^{21})^4\)

Determine whether each graph or table represents a linear or nonlinear function. Explain.

16. 

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>−11</td>
<td>18</td>
</tr>
<tr>
<td>−8</td>
<td>13</td>
</tr>
<tr>
<td>−5</td>
<td>8</td>
</tr>
<tr>
<td>−2</td>
<td>3</td>
</tr>
</tbody>
</table>

17. 

For Questions 18–20, graph each function.

18. \(y = \frac{1}{2}x^2 - 1\)

19. \(y = \frac{1}{3}x^3 + 1\)

20. \(y = 4^x - 3\)

B: ____________________

Bonus  Write all of the prime numbers between 1 and 50.

B: ____________________
Chapter 9 Extended-Response Test

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. Methods of finding prime numbers have intrigued mathematicians for years. For example, Goldbach’s conjecture states that every even number greater than 2 can be written as the sum of two prime numbers. Choose three even numbers greater than 20 and less than 100. Write each as the sum of two prime numbers.

2. Write an argument or counter example to support your answers to the following questions.
   a. \( \frac{x}{x} = 1 \) for all \( x \) except \( x = 0 \)
   b. \( \frac{x}{x+1} = 0 \) for all \( x \) except \( x = 1 \)

3. To understand mathematics, you must understand the language or symbols used.
   a. Explain the difference in the meanings of 3a and \( a^3 \).
   b. Explain the difference in the meanings of \(-3b\) and \( b^{-3} \).

4. Refer to the table at the right.
   a. Graph the ordered pairs in the table.
   b. Sketch a line or curve through the points.
   c. Does your graph represent a **linear** or a **nonlinear** function? Explain.
   d. Write a function that includes all of the ordered pairs in the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-7</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
</tbody>
</table>
Part 1: Multiple Choice

Instructions: Fill in the appropriate oval for the best answer.

1. Find the quotient of $3\frac{3}{4}$ and $\frac{5}{7}$. (Lesson 3-4)
   - A $5\frac{1}{4}$
   - B $3\frac{1}{28}$
   - C $2\frac{2}{3}$
   - D $\frac{4}{21}$

2. Simplify $(7k + 5) + 9k$. (Lesson 1-3)
   - F $16k + 35$
   - G $10k + 12$
   - H $16k + 5$
   - J $17k + 5$

3. Simplify $(-4)(3a)(-4b)$. (Lesson 2-4)
   - A $48ab$
   - B $-48ab$
   - C $-11ab$
   - D $-5ab$

4. Evaluate the expression $\frac{x}{y}$ if $x = -60$ and $y = 5$. (Lesson 2-5)
   - F $12$
   - G $\frac{1}{12}$
   - H $-\frac{1}{12}$
   - J $-12$

5. Name the ordered pair for the point A graphed on the coordinate plane at the right. (Lesson 2-6)
   - A $(3, 4)$
   - B $(3, -4)$
   - C $(4, 3)$
   - D $(4, -3)$

6. In the school cafeteria, an apple costs $a$ cents and a carton of milk costs 40 cents. Which expression represents the total cost of an apple and 2 cartons of milk for $n$ students? (Lesson 4-1)
   - F $a + 80 + n$
   - G $n(a + 80)$
   - H $na + 80$
   - J $n(a + 40)$

7. Wanda drove for $w$ hours on a trip. Her husband drove 3 hours more than Wanda. Which expression represents the total time they spent driving? (Lesson 4-2)
   - A $w + 6$
   - B $2w + 3$
   - C $w - 6$
   - D $2w - 3$

8. If $z - 5 = -9$, find the numerical value of $-2z - 5$. (Lesson 4-3)
   - F $23$
   - G $3$
   - H $-13$
   - J $-33$

9. Nine more than eight times a number is $-47$. Translate this sentence into an equation. (Lesson 4-6)
   - A $8n + 9 = -47$
   - B $9n + 8 = -47$
   - C $8(n + 9) = -47$
   - D $9(8 + n) = -47$

10. Find the slope of the line. (Lesson 8-5)
    - F $4$
    - G $\frac{1}{4}$
    - H $-\frac{1}{4}$
    - J $-4$
11. A book that normally sells for $3.95 is on sale for $2.37. What is the percent of discount on the book? (Lesson 7-5)  
A 25%  B 40%  C 42%  D 60%  

12. Write \((a)(a)(b)(b)\) using exponents. (Lesson 9-1)  
F \(a^3b^2\)  G \(a^{-3}b^{-2}\)  H \(3a^b\)  J \(3a^2b^2\)  

13. Write the prime factorization of 42. (Lesson 9-2)  
A \(1 \cdot 42\)  B \(2 \cdot 21\)  C \(2 \cdot 3 \cdot 7\)  D \(6 \cdot 7\)  

14. Which equation represents a linear function? (Lesson 9-7)  
F \(4xy = 15\)  G \(y = \frac{1}{3}x\)  H \(x^3 - 1 = y\)  J \(y = x \cdot (x + 7)\)  

15. Find the slope of the line through the points \((2, 4)\) and \((3, -6)\). (Lesson 8-5)  
A 10  B \(-\frac{1}{10}\)  C \(-\frac{5}{2}\)  D \(-10\)  

16. Which is \(\frac{1}{144}\) written with a negative exponent? (Lesson 9-4)  
F \(\frac{1}{12^2}\)  G \(12^{-2}\)  H \(14^{-2}\)  J \(144^{-2}\)  

17. Order the integers \({32, -18, 2, 7, 0, -5, -11}\) from least to greatest. (Lesson 2-1)  
A \{-5, -11, -18, 0, 2, 7, 32\}  C \{32, 7, 2, 0, -5, -11, -18\}  
B \{-18, -11, -5, 0, 2, 7, 32\}  D \{0, 2, -5, 7, -11, -18, 32\}  

18. Saturn is about 799,800,000 miles from Earth. Write this number in scientific notation. (Lesson 9-5)  
F \(7.998 \cdot 10^8\)  H \(0.7998 \cdot 10^9\)  
G \(79.98 \cdot 10^7\)  J \(7.998 \cdot 10^5\)  

Part 2: Gridded Response

Instructions: Enter your answers by writing each digit of the answer in a column box and then shading in the appropriate circle that corresponds to that entry.

19. The end zone of a football field is 30 feet wide and 160 feet long. What is its area in square feet? (Lesson 5-1)  

20. If \(f(x) = \frac{3}{4} x + 5\), find the value of \(f(7)\). (Lesson 7-1)
21. State the domain and range of the relation 
{(2.3, 4), (5, 3.2), (4.6, 3.3)}. (Lesson 1-4)

22. \(-7m + (-15m)\) (Lesson 1-2)

23. \(9x - (-23x)\)

24. Evaluate \(-\frac{4h}{3k}\) if \(h = 6\) and \(k = -2\). (Lesson 2-3)

25. Find the average (mean) of \(-24, 16, 21, 9, -12\). (Lesson 2-5)

26. Name the quadrant in which the graph of \((6, -5)\) lies. (Lesson 2-6)

27. When you divide a number by \(-9\), the result is 18. Write and solve an equation to find the number. (Lesson 4-4)

28. \(n^4\) (Lesson 9-1)

29. \(15r^3\)

30. Write the prime factorization of 78. (Lesson 9-2)

31. Wei-Ling wants to start a lawn in an area that is 110 feet by 70 feet. One bag of seed covers 1200 square feet. How many bags should he buy? (Lesson 5-1)

32. Write 2,850,000,000 in scientific notation (Lesson 9-5)

33. Find the slope of a line that passes through the points \(A(-2, 6)\) and \(B(5, -1)\). (Lesson 8-5)

34. Write the equation of the line that passes through the points \(C(-2, 1)\) and \(D(-1, 4)\) (Lesson 8-6)

35. The number of dogs at the humane society is increasing each month. The first month there are 20 dogs, the second month, there are 24 dogs, the third month, there are 28 dogs, and so on. (Lesson 8-2)
   
   a. Write an equation to represent this sequence.
   
   b. How many dogs will there be at the humane society after 6 months?
9 Unit 3 Test

(Chapters 8 and 9)

1. The table represents the temperature \( y \) at \( x \) kilometers above sea level for a certain location. Describe the relationship between the temperature and the altitude. Is this relation a function?

<table>
<thead>
<tr>
<th>Altitude (km)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°C)</td>
<td>24.0</td>
<td>17.4</td>
<td>10.8</td>
<td>4.2</td>
</tr>
</tbody>
</table>

2. Find the 16th term of the sequence 12, 18, 24, 30, … .

3. Find two solutions of \( 6x - 12y = 0 \). Write the solutions as ordered pairs.

4. Find the \( x \)-intercept and \( y \)-intercept of \( x - y = 2 \). Then graph the equation.

5. Find the slope of the line that passes through \( A(-7, 2) \) and \( B(3, -3) \).

6. The table shows the wind chill temperature for different wind speeds when the actual temperature is 25°F. State the rate of change.

<table>
<thead>
<tr>
<th>Wind (mph)</th>
<th>Temperature (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>( y )</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td>20</td>
<td>11</td>
</tr>
</tbody>
</table>

7. Write an equation in slope-intercept form for the line graphed.

8. If \((2, b)\) is on the graph of the line with slope \(-\frac{3}{2}\) and \(y\)-intercept 3, find the value of \( b \).

9. The scatter plot at the right shows a correlation between \( x \) and \( y \). Draw a best-fit line for the data. Use the best-fit line to predict the value of \( y \) when \( x \) is 4.
10. Find the product of \((3^6)(3)(3^5)\). Express your answer using exponents.

11. Evaluate \(3x^3\) if \(x = 4\).

12. Evaluate \(3r^3 + 4s\) if \(r = 2\) and \(s = 5\).

13. Write the prime factorization of 144.

14. Find \((3m^3n)(7mn)\).

15. Find \(\frac{a^{18}}{a^5}\).

16. Write \(5^{-7}\) using a positive exponent.

17. Express 25,400,500 in scientific notation.

18. Order \(4.1 \times 10^{-3}\), 0.041, 4100, \(4.1 \times 10^5\), 41,000 and \(1.4 \times 10^{-3}\) from least to greatest.

19. Simplify \((-4t^3)^3\).

20. Determine whether \(y = 0.4x^2\) is a linear or nonlinear equation. Explain.

21. Graph \(y = x^2 - 2\).

22. Graph \(y = -2x^3 + 1\).

23. Graph \(y = 3^x - 3\).
## 9 Anticipation Guide
### Powers and Nonlinear Functions

#### Step 1
Before you begin Chapter 9
- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

### Table: Step 1

<table>
<thead>
<tr>
<th>Statement</th>
<th>STEP 2 A or D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. In the expression $4^3$, the exponent is 4.</td>
<td>D</td>
</tr>
<tr>
<td>2. To evaluate $5^4$, multiply $5 \cdot 5 \cdot 5 \cdot 5$.</td>
<td>A</td>
</tr>
<tr>
<td>3. When simplifying expressions evaluate the powers last.</td>
<td>D</td>
</tr>
<tr>
<td>4. A prime number is a number that has exactly two factors, 1 and itself.</td>
<td>A</td>
</tr>
<tr>
<td>5. Multiply powers with the same base by multiplying their exponents.</td>
<td>D</td>
</tr>
<tr>
<td>6. $x^3$ is equivalent to $\frac{1}{x^3}$.</td>
<td>A</td>
</tr>
<tr>
<td>7. The number $23.4 \times 10^6$ is written in scientific notation.</td>
<td>D</td>
</tr>
<tr>
<td>8. $(ab)^m = ab^m$, for all numbers $a$ and $b$ and any integer $m$.</td>
<td>D</td>
</tr>
<tr>
<td>9. Nonlinear functions do not have constant rates of change.</td>
<td>A</td>
</tr>
<tr>
<td>10. The graph of a cubic function is called a parabola.</td>
<td>D</td>
</tr>
</tbody>
</table>

#### Step 2
After you complete Chapter 9
- Reread each statement and complete the last column by entering an A (Agree) or a D (Disagree).
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a separate sheet of paper to explain why you disagree. Use examples, if possible.

## 9-1 Study Guide and Intervention
### Powers and Exponents

#### Use Exponents
A number that is expressed using an exponent is called a **power**. The **base** is the number that is multiplied. The **exponent** tells how many times the base is used as a factor.

- **Definition**: $a^2$ has a base of 4 and an exponent of 3, and $4^3 = 4 \cdot 4 \cdot 4 = 64$.

- **Example**: Write each expression using exponents.
  a. $10 \cdot 10 \cdot 10 \cdot 10$
    - The base is 10. It is a factor 5 times, so the exponent is 5.
    - $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^5$
  b. $(-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9)$
    - The base is 9. It is a factor 6 times, so the exponent is 6.
    - $(-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9) = (-9)^6$
  c. $(p + 2)(p + 2)(p + 2)$
    - The base is $p + 2$. It is a factor 3 times, so the exponent is 3.
    - $(p + 2)(p + 2)(p + 2) = (p + 2)^3$

#### Exercises
Write each expression using exponents.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Step 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$</td>
<td>5</td>
</tr>
<tr>
<td>$-7 \cdot (-7) \cdot (-7)$</td>
<td>$(-7)^3$</td>
</tr>
<tr>
<td>$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$</td>
<td>$4^5$</td>
</tr>
<tr>
<td>$8 \cdot 8 \cdot 8$</td>
<td>$(-2)^6$</td>
</tr>
<tr>
<td>$(-2) \cdot (-2) \cdot (-2) \cdot (-2)$</td>
<td>$(-2)^4$</td>
</tr>
<tr>
<td>$6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$</td>
<td>$6^5$</td>
</tr>
<tr>
<td>$0.4 \cdot 0.4 \cdot 0.4$</td>
<td>$0.4^3$</td>
</tr>
<tr>
<td>$d \cdot d \cdot d \cdot d$</td>
<td>$d^4$</td>
</tr>
<tr>
<td>$m \cdot m \cdot m \cdot m \cdot m$</td>
<td>$m^5$</td>
</tr>
<tr>
<td>$x \cdot x \cdot y \cdot y$</td>
<td>$x^2y^2$</td>
</tr>
<tr>
<td>$(z - 4)(z - 4)$</td>
<td>$(z - 4)^2$</td>
</tr>
<tr>
<td>$3(-t)(-t)(-t)$</td>
<td>$3(-t)^3$</td>
</tr>
</tbody>
</table>
9-1 Study Guide and Intervention (continued)

Powers and Exponents

**Evaluate Expressions** When evaluating expressions with exponents you must follow the order of operations.

**Order of Operations**
1. Simplify expressions inside grouping symbols.
2. Evaluate all powers.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

**Example 1**

**ART** An artist is painting a mural that will look like a quilt square. The mural will have an area of $8^2$ square feet. How many square feet is this?

$8^2 = 8 \cdot 8$

$= 64$

Simplify.

The area of the mural will be 64 square feet.

**Example 2**

Evaluate $x^2 - 4$ if $x = -6$.

$x^2 - 4 = (-6)^2 - 4$

Replace $x$ with $-6$.

$= (-6)(-6) - 4$

$= 36 - 4$

$= 32$

Subtract.

Exercises

Evaluate each expression.

1. $7^2$
2. $3^5$
3. $(-6)^3$
4. $\frac{1}{5}$

5. $(-4)^2$
6. $2^8$
7. $3^3 - 6$
8. $8^2 - 9$

$= -121$

$= 729$

$= 500$

$= 256$

$= 162$

$= 11$

$= -384$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$

$= 8^2 \cdot 8^2$

$= 32 \cdot 32$

$= 1024$

$= 162$

$= 32$

$= 256$

$= 12$
9-1 Practice
Powers and Exponents

Write each expression using exponents.

1. $11 \cdot 11 \cdot 11$  
2. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$  
3. $5^4$  
4. $-4(-4) (-4)^2$  
5. $a \cdot a \cdot a \cdot a$  
6. $n \cdot n \cdot n \cdot n \cdot n$  
7. $4 \cdot 4 \cdot 4 \cdot 4$  
8. $(b \cdot b)(b \cdot b)$  
9. $(-v)(-v)(-v)(-v)$  
10. $x \cdot x \cdot x \cdot z \cdot z$  
11. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$  
12. $m \cdot m \cdot m \cdot p \cdot p$  
13. $(-6)(-6)(-6)(-6)$  
14. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot q \cdot q \cdot q$  
15. $y^6$  
16. $x^8$  
17. $y^4 - 8$  
18. $51^p$  
19. $z^2 16$  
20. $z^2$  
21. $9^729$  
22. $z^5 \cdot 2^6$  
23. $y^5 - 32$  
24. $z^3 - y^2$  
25. $x^3 + y^3 + z^3$  
26. $x^3 - z^3$  

Family Tree

For Exercises 27 and 28, refer to the following information.

When examining a family tree, the branches are many. You are generation “now.” One generation ago, your 2 parents were born. Two generations ago, your 4 grandparents were born.

27. How many great-grandparents were born three generations ago? $2^4$ or 8

28. How many “great” grandparents were born ten generations ago? $2^{10}$ or 1024

9-1 Word Problem Practice
Powers and Exponents

1. GEOMETRY Mr. Daniels is building a clubhouse for his children. He has decided that the floor will be a square with an area of 64 square feet. Write this number using a power greater than 1 and a lesser base. $8^2$

2. STOCK MARKET The Nikkei 225 is a stock market index that records the progress of 225 Japanese companies. Write this number using a power greater than 1 and a lesser base. $15^2$

3. NUMBER SENSE A googol is a very large number expressed as $10^{100}$. Ms. Rogers asked her students to determine which number is larger, a googol or $10^{50}$. Explain how her students might use the idea of repeated factors in order to find the solution. $10^{100}$ is 10 multiplied by itself 100 times. That would be the same as 1 followed by 100 zeros. $10^{50}$ is 100 multiplied by itself 10 times. That would be the same as 1 followed by 20 zeros. So, $10^{100}$ is larger.

4. LIFE SCIENCE A scientist is studying bacterial growth in the laboratory. She starts her experiment with 1 bacterium and then counts the bacteria at regular intervals and records them in the table below. If the pattern continues, how long will it take to have over 1000 bacteria?

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of cells</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

5. GEOMETRY The sides of right triangles have a special relationship. The longest side of a right triangle, always located opposite the right angle, is related to the shorter side lengths by the formula $c = \sqrt{a^2 + b^2}$, where $c$ is the length of the longest side and $a$ and $b$ are the lengths of the sides that intersect to form the right angle.

a. The following diagram shows a ladder leaning against a wall. The bottom of the ladder is 5 feet from the base of the wall, and the ladder reaches 12 feet up the wall. Find the length of the ladder. $\sqrt{169}$ or 13 ft

b. Paula exercises regularly by power walking around a rectangular field. She usually begins at one corner of the field and walks the full perimeter. One day, she takes a shortcut home by walking across the diagonal of the field. How far does she walk across the field? $\sqrt{2500}$ or 50 yd
9-1 Enrichment

Exponents
Numbers can be expressed in several ways. Some numbers are expressed as sums. Some numbers are expressed as products of factors, while other numbers are expressed as powers.
Two ways to express 27 are 3 · 3 · 3 and 3³.
The number 1 million can be expressed in the following ways.

\[
\begin{array}{cccc}
1,000,000 & 1,000 & 100 & 100 \cdot 100 \cdot 100 \\
10^6 & 10^3 & 10^2 \cdot 10^2 \cdot 10^2 & 10^3 \\
\end{array}
\]

Write names for each number below using the given exponents.
1. 16: exponents 2 and 4 4², 2⁴
2. 81: exponents 2 and 4 9², 3⁴
3. 64: exponents 2 and 6 8², 2⁶
4. 256: exponents 2 and 8 16², 2⁸
5. 625: exponents 2 and 4 25², 5⁴
6. 729: exponents 2 and 6 27², 3⁶
7. 2401: exponents 2 and 4 49², 7⁴
8. 4096: exponents 2 and 12 64², 2¹²
9. 6561: exponents 2 and 8 8¹, 2⁸
10. 390,625: exponents 2 and 8 625², 5⁸

Numbers that can be named as powers with like bases can be multiplied by adding the exponents.

\[
8 \cdot 8 = 2^3 \cdot 2^3 = 2^{3+3} = 2^6
\]

Write the product of each pair of factors in exponential form.
11. 9 · 9 = 3² · 3² = 3⁴
12. 4 · 4 = 2² · 2² = 2⁴
13. 16 · 8 = 2³ · 2³ = 2⁶
14. 125 · 25 = 5³ · 5³ = 5⁶
15. 27 · 9 = 3³ · 3² = 3⁵
16. 81 · 27 = 3⁴ · 3³ = 3⁷
17. 49 · 49 = 7² · 7² = 7⁴
18. 121 · 121 = 11³ · 11³ = 11⁶

Chapter 9 10
Glencoe Pre-Algebra

9-1 Graphing Calculator Activity

Evaluating Expressions with Exponents

There are several ways to evaluate expressions containing exponents. A base may be raised to any power using ^, the exponent key. Squares can be evaluated using ^2. And cubes can be evaluated using the cube command, which is item 3 in the MATH menu.

Example 1 Evaluate each expression.

a. 5²
Use the ^ key. Keystrokes: 5 ENTER

\[5^2 = 25\]

You can also use 5 2 ENTER.

b. (–4)³
Use the cube command. Keystrokes: 4 ENTER MATH 3 ENTER

\[(-4)^3 = -64\]

You can also use 4 4 ENTER 3 ENTER.

c. –2²
No parentheses are necessary since the expression is the opposite of two to the eighth power.
Keystrokes: 2 ENTER –2 ENTER = \(-2^2 = -4\)

Example 2 Evaluate each expression if \(a = 3\), \(b = 4\), and \(c = -5\).

Store the value of each variable.
Keystrokes: 3 4 5 ENTER ALPHA A ENTER 4 5 ENTER ALPHA B ENTER 3 5 ENTER ALPHA C ENTER

a. \(a^2/b/c\)

Keystrokes: ALPHA A ENTER 2 ALPHA B ENTER 5 ENTER ALPHA C ENTER

\[\frac{a^2}{b/c} = 280\]

b. \(b^4 - a^2 - c^6\)

Keystrokes: ALPHA B ENTER 4 ENTER 3 ALPHA A ENTER 2 ENTER ALPHA C ENTER 6 ENTER

\[\frac{b^4 - a^2 - c^6}{\text{ALPHA B ENTER} 2\text{ ENTER}} = 3372\]

\[\frac{2(3b + c)^3}{\text{ALPHA B ENTER} 2\text{ ENTER}} = 686\]

Exercises

Evaluate each expression.

1. \((-15)^2\) 225
2. \(-4^2 - 256\)
3. \(6^7 + 7776\)
4. \(9^3 - 729\)

Evaluate each expression if \(x = -2\), \(y = 6\), and \(z = -3\).

5. \(x^2y^2z^2\) 5184
6. \(z^2 - x^4 + y^2\) 209
7. \(12z^2 - x + 8\) 58
8. \(12(y - 5xz)^3 - 2592\)

Chapter 9 11
Glencoe Pre-Algebra
9-2 Study Guide and Intervention

Prime Factorization

Write Prime Factorizations A prime number is a whole number that has exactly two unique factors, 1 and itself. A composite number is a whole number that has more than two factors. Zero and 1 are neither prime nor composite.

Example 1 Determine whether each number is prime or composite.

a. 29

The only factors of 29 are 1 and 29, so 29 is a prime number.

b. 39

Find the factors of 39 by listing whole number pairs whose product is 39.

39 x 1 = 39
13 x 3 = 39

The factors of 39 are 1, 3, 13, and 39. Since the number has more than two factors, it is a composite number.

Any composite number can be written as a product of prime numbers. A factor tree can be used to find the prime factorization.

To make a factor tree:
1. Write the number that you are factoring at the top.
2. Choose any pair of whole number factors of the number.
3. Continue to factor any number that is not prime.

Example 2 Find the prime factorization of 48.

48 is the number to be factored.

6 x 8

2 x 2 x 2 x 2

The factor tree is complete when there is a row of prime numbers.

The prime factorization of 48 is 2 · 2 · 2 · 2 · 3 or 2⁴ · 3.

Exercises

Determine whether each number is prime or composite.

1. 27
2. 131
3. 77
4. 25

composites: 27, 131, 77, 25

primes: 1

Factor each monomial.

a. 3g²h³

48 is the number to be factored.

6 x 8

2 x 2 x 2 x 2

The factor tree is complete when there is a row of prime numbers.

The prime factorization of 48 is 2 · 2 · 2 · 2 · 3 or 2⁴ · 3.

Exercises

Determine whether each number is prime or composite.

a. 29
b. 39

c. 46

composites: 29, 39, 46

primes: none

Factor each monomial.

a. 3g²h³

48 is the number to be factored.

6 x 8

2 x 2 x 2 x 2

The factor tree is complete when there is a row of prime numbers.

The prime factorization of 48 is 2 · 2 · 2 · 2 · 3 or 2⁴ · 3.

Exercises

Determine whether each number is prime or composite.

a. 29
b. 39

c. 46

composites: 29, 39, 46

primes: none

Factor each monomial.

a. 3g²h³

48 is the number to be factored.

6 x 8

2 x 2 x 2 x 2

The factor tree is complete when there is a row of prime numbers.

The prime factorization of 48 is 2 · 2 · 2 · 2 · 3 or 2⁴ · 3.
Lesson 9-2

Prime Factorization

Determine whether each number is prime or composite.

1. 11 prime
2. 63 composite
3. 103 prime
4. 70 composite
5. 19 prime
6. 69 composite
7. 39 composite
8. 53 prime

Write the prime factorization of each number. Use exponents for repeated factors.

9. 39 = 3 × 13
10. 47 prime
11. 72 = 2^3 × 3^2
12. 276 = 2^2 × 3 × 23
13. 48 = 2^4 × 3
14. 1024 = 2^10
15. 95 = 5 × 19
16. 200 = 2^3 × 5^2
17. 90 = 2 × 3 × 5 × 3
18. 36 = 2^2 × 3^2
19. 32 = 2^5
20. 16 = 2^4
21. 16 = 2^4
22. 16 = 2^4
23. 16 = 2^4
24. 16 = 2^4
25. 16 = 2^4
26. 16 = 2^4
27. 16 = 2^4
28. 16 = 2^4
29. 16 = 2^4
30. 16 = 2^4
31. 16 = 2^4
32. 16 = 2^4

Answers

1. 11 prime
2. 63 composite
3. 103 prime
4. 70 composite
5. 19 prime
6. 69 composite
7. 53 prime
8. 39 composite

Write the prime factorization of each number. Use exponents for repeated factors.

9. 39 = 3 × 13
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19. 32 = 2^5
20. 16 = 2^4
21. 16 = 2^4
22. 16 = 2^4
23. 16 = 2^4
24. 16 = 2^4
25. 16 = 2^4
26. 16 = 2^4
27. 16 = 2^4
28. 16 = 2^4
29. 16 = 2^4
30. 16 = 2^4
31. 16 = 2^4
32. 16 = 2^4

NUMBER THEORY

Twin primes are a pair of consecutive odd primes, which differ by 2. For example, 3 and 5 are twin primes. Find the twin primes less than 100.

41, 43; 59, 61; 71, 73
9-2 Word Problem Practice

Prime Factorization

1. ADDRESSES On the block where Liza lives, the house numbers of all the two-story homes are prime numbers. Liza’s block contains house numbers between 40 and 60. List all of the house numbers of the houses that you know have two stories. 41, 43, 47, 53, and 59

2. PACKAGING Renata’s tool cabinet is a rectangular box with a volume of 100 cubic feet. Find the dimensions (length, width, and height) of the cabinet if each dimension is a whole number greater than 1. 3 ft by 5 ft by 7 ft

3. CLASS PROJECT Amad and Lynette were researching prime numbers for a class project. Each discovered an expression relating to prime numbers. Amad found that all prime numbers greater than 2 can be expressed as 4e + 1 where e is a natural number. Lynette found that all primes greater than 3 can be expressed as 6b + 1 where b is a natural number. Find the values of e and b for the prime number 113. a = 28 and b = 19

4. LANDSCAPING A tiled area next to Gerard’s house has an area of 60 sq. ft. Length Width

Write two possible pairs of expressions that could represent the length and the width of the tiled area. Sample answer: 3x × 2t or 6 × x

5. BIRTHDAY Darnell was born in October. Rather than tell his friends on what day he was born, he tells them that his birthday has three distinct prime factors. On what day was Darnell born? October 30th

6. NUMBER THEORY In order to determine if a number is prime or composite, it is helpful to test whether prime numbers are factors of the number, starting with the lowest prime number.

David wants to determine whether or not the number 73 is a prime number. He has already determined that the prime numbers 2, 3, 5, and 7 are not factors of 73.

a. How does David know that 4 and 6 cannot be factors because 2 and 3 are not factors of 73?

b. Does David need to test any other numbers to see if 73 is a prime number? (Hint: \(7 \times 7 = 49\)) He does not need to test any composite numbers. The next larger prime after 7 is 11 and 11\(^2\) is 121, which is larger than 73. If all primes less than 11 are not factors of 73, then 73 is prime.

c. Use this method to determine if 437 is a prime number. List all the potential factors you tested. Test 2, 3, 5, 7, 11, 13, 17, and 19. It is not prime because \(19 \times 23 = 437\).

9-2 Enrichment

Prime Pyramid

A prime number is a whole number that has exactly two factors—itself and 1. The pyramid below is called a prime pyramid. Each row begins with 1 and ends with the number of that row. So, row 2 begins with 1 and ends with 2, row 3 begins with 1 and ends with 3, and so on. In each row, the numbers from 1 to the row number are arranged such that the sum of any two adjacent numbers is a prime number.

For example, look at row 4:

- It must contain the numbers 1, 2, 3, and 4.
- It must begin with 1 and end with 4.
- The sum of adjacent pairs must be a prime number:
  - \(1 + 2 = 3\), \(2 + 3 = 5\), \(3 + 4 = 7\)

1. Complete the pyramid by filling in the missing numbers.

2. Extend the pyramid to row 13.

3. Explain the patterns you see in the completed pyramid.

Sample answer: Each row alternates odd and even numbers. Multiples of 3 form diagonals that are constant.


## 9-3 Study Guide and Intervention

### Multiplying and Dividing Monomials

**Multiply Monomials**
When multiplying powers with the same base, add the exponents.

Symbols: \(a^m \cdot a^n = a^{m+n}\)

**Example**
\[4^2 \cdot 4^3 = 4^{2+3} = 4^5\]

### Example 1
Find each product.

a. \(5^2 \cdot 5\)
   \[5^2 \cdot 5 = 5^{2+1} = 5^3\]

b. \(7^3 \cdot 7^2\)
   \[7^3 \cdot 7^2 = 7^{3+2} = 7^5\]

**Exercises**
**Find each product. Express using exponents.**

1. \(4^3 \cdot 4^2\)
2. \(v^3 \cdot v^4\)
3. \((f^2)^3\)
4. \((-31^3) \cdot (-31^2)\)
5. \((-c^3)(-c^2)\)
6. \(22 \cdot 22^3\)
7. \(7a \cdot 5a^2\)
8. \(-10x \cdot 7x^2\)
9. \(5p \cdot (-4p)\)
10. \(35^4 \cdot -70^5\)
11. \((-14y) \cdot x\)
12. \(9x^2 \cdot 2x\)
13. \(36a^6 \cdot 14\)
14. \(-12u^3 (6u^2)\)
15. \(-5m^3 (4m^6)\)
16. \(31^3\)
17. \(42u^{11}\)

### Divide Monomials
When dividing powers with the same base, subtract the exponents.

Symbols: \(\frac{a^m}{a^n} = a^{m-n}\), where \(a \neq 0\)

**Example**
\[\frac{5^2}{5^3} = 5^{2-3} = 5^{-1}\]

### Example 1
Find each quotient.

a. \(-8^4 \div (-8)^2\)
   \[\frac{-8^4}{(-8)^2} = (-8)^{4-2} = (-8)^2\]

b. \(\frac{a^4}{a^3}\)
   \[\frac{a^4}{a^3} = a^{4-3} = a^1\]

### Example 2
RIVERS The Mississippi River is approximately \(3 \, \text{million}\) miles long. About how many times as long is the Mississippi River than the Kentucky River?

Write a division expression to compare the lengths.

\[\frac{3 \, \text{million}}{3^1} = 3 \, \text{million}\]

So, the Mississippi River is approximately 9 times as long as the Kentucky River.

### Exercises
**Find each quotient. Express using exponents.**

1. \(\frac{7^3}{7^1}\)
2. \(\frac{3^3}{3^1}\)
3. \(\frac{-12^3}{-12^1}\)
4. \(\frac{c^8}{c^3}\)
5. \(\frac{-p^9}{-p^3}\)
6. \(\frac{2u^2}{2u}\)
7. \(\frac{c^4}{c}\)
8. \(\frac{k^8}{k}\)
9. \(3v^2 + 3v\)
10. \(12x^3 + 12x^3\)
11. \((-2a^2) + (-2a)\)
12. \(5x^2 + 5y^3\)
13. \(v^2\)
14. \(x^4\)
15. \((9^a)^{1/3}\)
16. \((f)^{1/2}\)
9-3 Skills Practice

Multiplying and Dividing Monomials

Find each product or quotient. Express using exponents.

1. \(2^2 \cdot 2^3\)  
2. \(10^4 \cdot 10^7\)  
3. \(1^4 \cdot 1^5\)  
4. \(6^4 \cdot 6^6\)  
5. \((-3)^3 \cdot (-3)^2\)  
6. \((-9)^3 \cdot (-9)^4\)  
7. \(a^2 \cdot a^3\)  
8. \(n^4 \cdot n^2\)  
9. \((p^3)^4\)  
10. \((x^2)^3\)  
11. \((6^3)^2\)  
12. \((-v)^6 \cdot (-v)^7\)  
13. \(11a^2 \cdot 3a^3\)  
14. \(10t^3 \cdot 4t^2\)  
15. \((8c^6)^3\)  
16. \((4j^8)^3\)  
17. \(5^5 \cdot 5^2\)  
18. \(10^5 \cdot 10^6\)  
19. \(\frac{y^4}{y^2} \cdot y^3\)  
20. \(12^3 \cdot 12^2\)  
21. \(\frac{100^7}{100^6}\)  
22. \(\frac{(-2)^4}{-2} \cdot (-2)^2\)  
23. \(r^2 \cdot r^3\)  
24. \(\frac{z^4}{z^2}\)  
25. \(\frac{q^3}{q^4}\)  
26. \(\frac{g^5}{g^6}\)  
27. \(\frac{(-y)^5}{(-y)^2}\)  
28. \(\frac{(-z)^9}{(-z)^2}\)  
29. the product of two squared and two to the sixth power  
30. the quotient of ten to the seventh power and ten cubed  
31. the product of \(y\) squared and \(y\) cubed  
32. the quotient of \(a\) to the twentieth power and \(a\) to the tenth power  

NAME _____________________________ DATE _____________ PERIOD _____________

Answer Key

1. \(2^5\)  
2. \(10^{11}\)  
3. \(1^9\)  
4. \(6^{10}\)  
5. \((-3)^5\)  
6. \((-9)^7\)  
7. \(a^5\)  
8. \(n^7\)  
9. \(p^12\)  
10. \(x^{13}\)  
11. \(18b^7\)  
12. \((-v)^{10}\)  
13. \(33a^8\)  
14. \(40t^{12}\)  
15. \(72c^5\)  
16. \(20f^{14}\)  
17. \(5^2\)  
18. \(10^6\)  
19. \(\frac{y}{z}\)  
20. \(12^5\)  
21. \(10^7\)  
22. \((-2)^2\)  
23. \(r^5\)  
24. \(z^2\)  
25. \(q^4\)  
26. \(g^4\)  
27. \((-y)^6\)  
28. \((-z)^3\)  
29. \(5^7\)  
30. \(10^{14}\)  
31. \(y^5\)  
32. \(a^{10}\)  

Chapter 9

NAME _____________________________ DATE _____________ PERIOD _____________

9-3 Practice

Multiplying and Dividing Monomials

Find each product or quotient. Express using exponents.

1. \(4^4 \cdot 4^5\)  
2. \(9^9 \cdot 9^{14}\)  
3. \(7^7 \cdot 7^8\)  
4. \(13^3 \cdot 13^4\)  
5. \((-8)^3 \cdot (-8)^6\)  
6. \((-21)^2 \cdot (-21)^7\)  
7. \(f^4 \cdot f^2\)  
8. \(h^4 \cdot h^7\)  
9. \((m^4)^3\)  
10. \((w^3)^5\)  
11. \((-r)^3 \cdot (-r)^6\)  
12. \((w)^{10} \cdot (w)^{10}\)  
13. \(6^4 \cdot 8d^7\)  
14. \(7^6 \cdot 6^2\)  
15. \(-50^6 \cdot 6^3\)  
16. \(12^3 \cdot 12^2\)  
17. \(6^1 \cdot 6^4\)  
18. \(5^5 \cdot 5^1\)  
19. \(9^6 \cdot 9^2\)  
20. \(18^8\)  
21. \((-7)^7 \cdot (-7)^1\)  
22. \(95^7\)  
23. \(a^{11} \cdot a^{10}\)  
24. \(a^{11} \cdot a^{10}\)  
25. the product of five cubed and five to the fourth power  
26. the quotient of eighteen to the ninth power and eighteen squared  
27. the product of \(z\) cubed and \(z\) cubed  
28. the quotient of \(x\) to the fifth power and \(x\) cubed  
29. \(10^7\) or 1,000,000 times  
30. \(10^7\) or 1,000,000 times  
31. \(10^7\) or 1,000,000 times  
32. \(10^7\) or 1,000,000 times  

Chapter 9

20

Glencoe Pre-Algebra

Chapter 9

21

Glencoe Pre-Algebra
9-3 Word Problem Practice

Multiplying and Dividing Monomials

1. BIOLOGY Ms. Masse's biology class is conducting an experiment to record the growth of a certain kind of bacteria. Each student has a lab dish containing 2 bacteria which are able to double every day. How many bacteria will be present in a student's lab dish after two weeks? 16,384

2. COMPUTERS In 1995, the average home computer had a speed of about 10⁷ cycles per second. In 2004, the average home computer had a speed of 10¹⁰ cycles per second. How many times faster were the computers in 2004 as compared to those in 1995? 1,000 times faster

3. CATERING A gourmet meal catering company is planning an event for 2³ people. One week before the event, they find out that the number of people has doubled. Will there be 2⁴ or 2⁶ people at the event? Explain. Sample answer: There will be 2⁴ people at the event because multiplying 2³ by two is the same as multiplying 2⁴ itself eight times, or 2⁶.

4. HOMEWORK Vance and Ko are trying to simplify the expression 3³ × 3⁵. Their answers are different:
   Ko's work: 3³ × 3⁵ = 3²³ = 9³
   Vance's work: 3³ × 3⁵ = 3⁸ = 3²

Which student is correct? Identify the mistake made by the other student. Sample answer: Vance is correct. Ko should not have multiplied the base by itself.

5. SOUND Levels of audible sound are measured in decibels (dB). An increase in 10dB is considered a doubling (2¹) of perceivable sound to the human ear. The table below lists the decibel level of some common sounds.

<table>
<thead>
<tr>
<th>Sound Level (dB)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>Library, no talking</td>
</tr>
<tr>
<td>70</td>
<td>Busy traffic</td>
</tr>
<tr>
<td>80</td>
<td>Hair dryer</td>
</tr>
<tr>
<td>100</td>
<td>Garbage truck</td>
</tr>
<tr>
<td>110</td>
<td>Rock concert</td>
</tr>
<tr>
<td>130</td>
<td>Jet engine</td>
</tr>
<tr>
<td>180</td>
<td>Rocket launching pad</td>
</tr>
</tbody>
</table>

Source: Seattle Dept. of Planning and Development

a. Karen was walking on a sidewalk along a road with busy traffic. She noticed that a garbage truck seemed much louder than the traffic. How many times louder was the garbage truck than the busy traffic? 8 times louder

b. Most people do not wear hearing protection when drying their hair. However, airport workers often wear ear protection because of the sound produced by jet engines. How many times louder is a jet engine than a hair dryer? 16 times louder

c. How many times louder is it at a rocket launching pad than in a library? 32,768 times louder

9-3 Enrichment

Dividing Powers with Different Bases

Some powers with different bases can be divided. First, you must be able to write both as powers of the same base. An example is shown below.

\[
\frac{2^4}{2^2} = \frac{2^1}{2^3}
\]

To find the power of a power, multiply the exponents.

\[
\frac{2^1}{2^3} = 2^{-1} \text{ or } \frac{1}{2}
\]

This method could not have been used to divide \( \frac{2^1}{9^2} \), since 9 cannot be written as a power of 2 using integers.

Simplify each fraction using the method shown above. Express the solution without exponents.

1. \( \frac{8^1}{2^2} \)
2. \( \frac{16^9}{8^3} \)
3. \( \frac{9^1}{3^2} \)
4. \( \frac{512^3}{3^1} \)
5. \( \frac{67^1}{8^1} \)
6. \( \frac{36^1}{16^1} \)
7. \( \frac{125^1}{25^1} \)
8. \( \frac{6^2}{216^1} \)
9. \( \frac{30^1}{1000^1} \)
10. \( \frac{64^1}{8^1} \)
11. \( \frac{27^2}{9^1} \)
12. \( \frac{343^1}{7^3} \)
9-4 Study Guide and Intervention

Negative Exponents

Extending the pattern below shows that $4^{-1} = \frac{1}{4}$ or $4 \cdot 4^{-1} = 4$.

<table>
<thead>
<tr>
<th>Example</th>
<th>Expression</th>
<th>Definition of Negative Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. $3^{-4}$</td>
<td>$\frac{1}{3^4}$</td>
<td>$a^{-n} = \frac{1}{a^n}$ for $a \neq 0$ and any whole number $n$. Example: $6^{-4} = \frac{1}{6^4}$</td>
</tr>
<tr>
<td>b. $y^{-4}$</td>
<td>$\frac{1}{y^4}$</td>
<td>Definition of negative exponent</td>
</tr>
</tbody>
</table>

For $a \neq 0$, $a^0 = 1$. Example: $9^0 = 1$

Example 2

Write each fraction as an expression using a negative exponent other than $-1$.

<table>
<thead>
<tr>
<th>Example</th>
<th>Expression</th>
<th>Definition of Negative Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. $\frac{1}{a^2}$</td>
<td>$6^{-2}$</td>
<td>$\frac{1}{a^n} = a^{-n}$. Definition of negative exponent</td>
</tr>
<tr>
<td>b. $\frac{1}{81}$</td>
<td>$\frac{1}{9^2}$</td>
<td>$\frac{1}{9^2} = 9^{-2}$. Definition of negative exponent</td>
</tr>
</tbody>
</table>

Exercises

Write each expression using a positive exponent.

1. $6^4 \cdot 8^2$
2. $(-7)^{-2} \cdot (-7)^2$
3. $b^{-2} \cdot b^2$
4. $n^{-1} \cdot \frac{1}{n}$ or $\frac{1}{n}$
5. $(-2)^{-1} \cdot (-2)^2$
6. $10^{-3} \cdot 10^7$
7. $j^{-3} \cdot \frac{1}{j^3}$
8. $a^{-1} \cdot a^3$

Write each fraction as an expression using a negative exponent other than $-1$.

9. $\frac{1}{b^2}$
10. $\frac{1}{12^2}$
11. $\frac{1}{25}$
12. $\frac{1}{49}$
13. $\frac{1}{3^2}$
14. $\frac{1}{9^2}$
15. $\frac{1}{121}$
16. $\frac{1}{27}$

Example 1
Evaluate $b^{-3}$ if $b = 3$.

Replace $b$ with 3.

$= \frac{1}{3}$

Example 2
Evaluate $8c^{-4}$ if $c = 2$.

Replace $c$ with 2.

$= \frac{8}{16}$

$= \frac{1}{2}$

Simplify.

Exercises
Evaluate each expression if $m = -4$, $n = 1$, and $p = 6$.

1. $p^{-3} \cdot \frac{1}{36}$
2. $m^{-3} \cdot \frac{1}{64}$
3. $(np)^{-1} \cdot \frac{1}{6}$
4. $3^{-2} \cdot \frac{1}{81}$
5. $p^{-2} \cdot \frac{1}{1296}$
6. $(2n)^{-2} \cdot \frac{1}{64}$
7. $m^{-3} \cdot \frac{1}{4096}$
8. $(mp)^{-1} \cdot \frac{1}{24}$
9. $4^{-1} \cdot \frac{1}{256}$
10. $-3^{-2} \cdot \frac{1}{3}$
11. $m^2 \cdot \frac{1}{9}$
12. $pm^{-3} \cdot \frac{3}{8}$
**9-4 Skills Practice**

**Negative Exponents**

Write each expression using a positive exponent.

1. \(3^{-2} \frac{1}{3^2}\)
2. \(2^{-1} \frac{1}{8}\)
3. \(10^{-1} \frac{1}{10}\)
4. \((-2)^{-2} \frac{1}{(-2)^2}\)
5. \((-40)^{-2} \frac{1}{(-40)^2}\)
6. \((-17)^{-2} \frac{1}{(-17)^2}\)
7. \(n^{-2} \frac{1}{n^2}\)
8. \(b^{-2} \frac{1}{b^2}\)
9. \(q^{-2} \frac{1}{q^2}\)
10. \(m^{-2} \frac{1}{m^2}\)
11. \(v^{-1} \frac{1}{v^1}\)
12. \(p^{-2} \frac{1}{p^2}\)

Write each fraction as an expression using a negative exponent other than \(-1\).

13. \(\frac{1}{8^2}\) \(-2\)
14. \(\frac{1}{10^2}\) \(-6\)
15. \(\frac{1}{2^2}\) \(-2\)
16. \(\frac{1}{6^2}\) \(-7\)
17. \(\frac{1}{17^2}\) \(-17\)
18. \(\frac{1}{21^2}\) \(-21\)
19. \(\frac{1}{3^2}\) \(-3\)
20. \(\frac{1}{9^2}\) \(-9\)
21. \(\frac{1}{3^2}\) \(-3\)
22. \(\frac{1}{12^2}\) \(-12\)
23. \(\frac{1}{25^2}\) \(-5\)
24. \(\frac{1}{36^2}\) \(-6\)

Evaluate each expression if \(x = 1, y = 2,\) and \(z = -3\).

25. \(y^{-1} 8\)
26. \(z^{-2} \frac{1}{9}\)
27. \(x^{-2} 1\)
28. \(y^{-1} \frac{1}{32}\)
29. \(z^{-3} - \frac{1}{27}\)
30. \(y^{-1} \frac{1}{2}\)
31. \(z^{-1} \frac{1}{81}\)
32. \(5^{-1} \frac{1}{125}\)
33. \(x^{-1} 1\)
34. \(1^{-1} 1\)
35. \(4^{-1} \frac{1}{64}\)
36. \(y^{-1} \frac{1}{8}\)

---

**9-4 Practice**

**Negative Exponents**

Write each expression using a positive exponent.

1. \(7^{-1} \frac{1}{7}\)
2. \(10^{-1} \frac{1}{10}\)
3. \(23^{-1} \frac{1}{23}\)
4. \((-5)^{-1} \frac{1}{(-5)^2}\)
5. \((-18)^{-1} \frac{1}{(-18)^2}\)
6. \(m^{-1} \frac{1}{m}\)
7. \((-1)^{-2} \frac{1}{(-1)^2}\)
8. \(c^{-2} \frac{1}{c^2}\)
9. \(p^{-2} \frac{1}{p^2}\)
10. \(g^{-1} \frac{1}{g}\)
11. \(5g^{-1} \frac{1}{5g}\)
12. \(3^{-1} \frac{1}{3}\)

Write each fraction as an expression using a negative exponent.

13. \(\frac{1}{2^3} 2^{-10}\)
14. \(\frac{1}{39} 39^{-1}\)
15. \(\frac{1}{4^3} 4^{-3}\)
16. \(\frac{1}{39} 39^{-1}\)
17. \(\frac{1}{81} 81^{-1}\)
18. \(\frac{1}{m^3} m^{-3}\)
19. \(\frac{1}{x^3} x^{-3}\)
20. \(\frac{1}{a^3} a^{-3}\)
21. \(\frac{1}{49} 7^{-2}\)
22. \(\frac{1}{8} 2^{-3}\)
23. \(\frac{1}{144} 12^{-2}\)
24. \(\frac{1}{169} 13^{-2}\)

Evaluate each expression if \(x = 3, y = -2,\) and \(z = 4\).

25. \(x^{-1} \frac{1}{81}\)
26. \(y^{-1} \frac{1}{4}\)
27. \(y^{-1} - \frac{1}{32}\)
28. \(x^{-1} \frac{1}{256}\)
29. \(z^{-1} \frac{1}{25}\)
30. \(10^{-1} \frac{1}{100}\)
31. \(3z^{-1} \frac{3}{4}\)
32. \(2z^{-1} \frac{1}{16}\)
33. \((xz)^{-1} \frac{1}{144}\)

34. **Hair** Hair grows at a rate of \(\frac{1}{64}\) inch per day. Write this number using negative exponents.

\(8^{-2}\) or \(4^{-3}\) or \(2^{-6}\)
**9-4 Word Problem Practice**

**Negative Exponents**

1. **SOLAR SYSTEM** The distance between Earth and the Sun is about \( \frac{1}{100,000} \) the diameter of the solar system. Express this number using a negative exponent other than \(-1\). \(10^{-6}\)

2. **PAPER** The paper used by the students at Hopkins Middle School is approximately \( \frac{1}{200} \) inch thick. Express this number using a negative exponent other than \(-1\). \(6^{-3}\)

3. **TIME** A microsecond is a measure of time that is equal to one millionth of a second. Express this number as a power of 10 with a negative exponent. \(10^{-6}\)

4. **MEASUREMENT** There are \(10^{-5}\) meters in 1 centimeter. At the site of an automobile accident, a state trooper uses a measuring tape to determine that the width of a tire track is 20 centimeters. Express this number as a fraction of a meter in simplest form. \(20 = \frac{1}{5} \text{ m}\)

5. **HOMEWORK** As Libby was working on her math homework, she computed \(2^{-3}\) by writing the following equation. \(2^{-3} = -8\)

   What was Libby's error? Explain. Then give the correct answer. \(2^{-3} \neq 0.125\); Sample answer: Libby took the opposite of \(2^3\) instead of taking the reciprocal of \(2^3\)

6. **INSECTS** Kevin's father is an entomologist. He studies insects. The table below shows the mass of four common insects.

<table>
<thead>
<tr>
<th>Insect</th>
<th>Mass (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Honeybee</td>
<td>8(^{-2})</td>
</tr>
<tr>
<td>Ant</td>
<td>10(^{-2})</td>
</tr>
<tr>
<td>Housefly</td>
<td>9(^{-2})</td>
</tr>
<tr>
<td>Moth</td>
<td>45(^{-2})</td>
</tr>
</tbody>
</table>

**Proving Definitions of Exponents**

Recall the rules for multiplying and dividing powers with the same base. Use these rules, along with other properties you have learned, to justify each definition. Abbreviations for some properties you may wish to use are listed below.

- Associative Property of Multiplication (APM)
- Additive Identity Property (AIP)
- Multiplicative Identity Property (MIP)
- Inverse Property of Addition (IPA)
- Inverse Property of Multiplication (IPM)

Write the reason for each statement.

1. Prove: \(a^0 = 1\)

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>Let (m) be an integer, and let (a) be any nonzero number. (a^m \cdot a^0 = a^{m+0})</td>
<td>a. Given</td>
</tr>
<tr>
<td>(a^m \cdot a^0 = a^m)</td>
<td>b. <strong>Product of Powers</strong></td>
</tr>
<tr>
<td>(\frac{1}{a^m \cdot a^0} = \frac{1}{a^m} \cdot a^0)</td>
<td>c. <strong>IPA</strong></td>
</tr>
<tr>
<td>(\frac{1}{a^m} \cdot a^0 = a^0)</td>
<td>d. <strong>Mult. Prop. Equality</strong></td>
</tr>
<tr>
<td>(1 \cdot a^0 = 1)</td>
<td>e. <strong>APM</strong></td>
</tr>
<tr>
<td>(a^0 = 1)</td>
<td>f. <strong>MIP</strong></td>
</tr>
</tbody>
</table>

2. Prove: \(a^{-1} = \frac{1}{a^1}\)

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>Let (a) be an integer, and let (a) be any nonzero number. (a^{-1} \cdot a^1 = a^{-1+1})</td>
<td>a. Given</td>
</tr>
<tr>
<td>(a^{-1} \cdot a^1 = a^0)</td>
<td>b. <strong>Product of Powers</strong></td>
</tr>
<tr>
<td>(a^{-1} \cdot a^1 = 1)</td>
<td>c. <strong>IPA</strong></td>
</tr>
<tr>
<td>((a^{-1} \cdot a^1) \cdot a = a^{-1} \cdot a^2)</td>
<td>d. <strong>Def. Zero Exponents</strong></td>
</tr>
<tr>
<td>(a^{-1} \cdot a^2 = a^{-1+2})</td>
<td>e. <strong>Mult. Prop. Equality</strong></td>
</tr>
<tr>
<td>(a^{-1} \cdot a^2 = a^{-1} \cdot a^0)</td>
<td>f. <strong>APM</strong></td>
</tr>
<tr>
<td>(a^{-1} \cdot a^2 = \frac{1}{a^1} \cdot a^0)</td>
<td>g. <strong>MIP</strong></td>
</tr>
<tr>
<td>(a^{-1} \cdot a^2 = \frac{1}{a^1})</td>
<td>h. <strong>MIP</strong></td>
</tr>
</tbody>
</table>
Scientific Notation

Numbers like 5,000,000 and 0.0005 are in standard form because they do not contain exponents. A number is expressed in scientific notation when it is written as a product of a factor and a power of 10. The factor must be greater than or equal to 1 and less than 10. By definition, a number in scientific notation is written as \( a \times 10^n \), where \( 1 \leq a < 10 \) and \( n \) is an integer.

Example 1
Express each number in standard form.

a. \( 6.32 \times 10^6 \)
   \[ 6.32 \times 10^6 = 6,320,000 \]
   Move the decimal point 6 places to the right.

b. \( 7.8 \times 10^{-4} \)
   \[ 7.8 \times 10^{-4} = 0.00078 \]
   Move the decimal point 4 places to the left.

Example 2
Express each number in scientific notation.

a. \( 62,000,000 \)
   To write in scientific notation, place the decimal point after the first nonzero digit, then find the power of 10.
   \[ 62,000,000 = 6.2 \times 10^7 \]
   The decimal point moves 7 places.
   Move the decimal point 1 place to the left.

b. \( 0.00025 \)
   \[ 0.00025 = 2.5 \times 10^{-4} \]
   The decimal point moves 4 places.
   Move the decimal point 1 place to the right.

Exercises
Express each number in standard form.

1. \( 4.12 \times 10^6 \)
2. \( 5.89 \times 10^4 \)
3. \( 6.4 \times 10^3 \)
4. \( 7.8 \times 10^2 \)
5. \( 8.06 \times 10^1 \)
6. \( 9.01 \times 10^0 \)
7. \( 10^1 \)
8. \( 10^2 \)
9. \( 10^3 \)
10. \( 10^4 \)
11. \( 10^5 \)
12. \( 10^6 \)
13. \( 10^7 \)
14. \( 10^8 \)
15. \( 10^9 \)
16. \( 10^{10} \)

Chapter 9

Scientific Notation

Compare and Order Numbers

You can compare and order numbers in scientific notation without converting them into standard form.

To compare numbers in Scientific Notation, compare the exponents.

- If the exponents are positive, the number with the greatest exponent is the greatest.
- If the exponents are negative, the number with the least exponent is the least.
- If the exponents are the same, compare the factors.

Example 1
Compare each set of numbers using <, >, or =.

a. \( 2.097 \times 10^5 \) \( \quad \) \( 3.12 \times 10^6 \)
   So, \( 2.097 \times 10^5 < 3.12 \times 10^6 \).

b. \( 8.706 \times 10^{-5} \) \( \quad \) \( 8.089 \times 10^{-5} \)
   The exponents are the same, so compare the factors.
   \( 8.706 < 8.089 \).

Example 2
ATOMS The table shows the weight of protons, neutrons, and electrons. Rank the particles in order from heaviest to lightest.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron</td>
<td>( 9.109 \times 10^{-3} )</td>
</tr>
<tr>
<td>Proton</td>
<td>( 1.672 \times 10^{-1} )</td>
</tr>
<tr>
<td>Neutron</td>
<td>( 1.674 \times 10^{-2} )</td>
</tr>
</tbody>
</table>

Step 1: Order the numbers according to their exponents. The electron has an exponent of \(-31\). So, it has the least weight.

Step 2: Order the numbers with the same exponent by comparing the factors.

So, \( 1.674 \times 10^{-2} < 1.672 \times 10^{-1} < 9.109 \times 10^{-3} \).

The order from heaviest to lightest is neutron, proton, and electron.

Exercises
Choose the greater number in each pair.

1. \( 4.9 \times 10^4 \) \( \quad \) \( 9.9 \times 10^{-4} \)
2. \( 2.004 \times 10^3 \) \( \quad \) \( 2.005 \times 10^2 \)
3. \( 3.2 \times 10^5 \) \( \quad \) \( 4.9 \times 10^6 \)
4. \( 0.002 \times 10^5 \) \( \quad \) \( 3.6 \times 10^2 \)

Order each set of numbers from least to greatest.

5. \( 6.0 \times 10^3 \) \( \quad \) \( 7.9 \times 10^2 \)
6. \( 6.8 \times 10^5 \) \( \quad \) \( 7.1 \times 10^4 \)
7. \( 8.4 \times 10^6 \) \( \quad \) \( 9.6 \times 10^5 \)
8. \( 4.0 \times 10^7 \) \( \quad \) \( 5.1 \times 10^6 \)
9. \( 8.1 \times 10^8 \) \( \quad \) \( 9.8 \times 10^7 \)
10. \( 8.2 \times 10^9 \) \( \quad \) \( 9.6 \times 10^8 \)
11. \( 8.1 \times 10^{10} \) \( \quad \) \( 8.1 \times 10^9 \)
12. \( 210,000,000 \) \( \quad \) \( 2.05 \times 10^{10} \)
13. \( 2.15 \times 10^{11} \) \( \quad \) \( 2.05 \times 10^{9} \)
14. \( 2.10,000,000 \) \( \quad \) \( 2.05 \times 10^{10} \)

Chapter 9

Glencoe Pre-Algebra
Skills Practice

Scientific Notation

Express each number in standard form.

1. $1.5 \times 10^6$ 1500
2. $4.01 \times 10^4$ 40,100
3. $6.78 \times 10^6$ 678
4. $5.925 \times 10^6$ 5,925,000
5. $7.0 \times 10^{12}$ 700,000,000
6. $9.99 \times 10^{12}$ 99,900,000
7. $3.0005 \times 10^6$ 300,050
8. $2.54 \times 10^5$ 254,000
9. $1.75 \times 10^{15}$ 17,500
10. $1.2 \times 10^{-8}$ 0.00000012
11. $7.0 \times 10^{-1}$ 0.7
12. $6.3 \times 10^{-9}$ 0.000000063
13. $5.83 \times 10^{-2}$ 0.0583
14. $8.075 \times 10^{-4}$ 0.0008075
15. $1.1 \times 10^{-3}$ 0.00011

Express each number in scientific notation.

17. $1,000,000$ $1.0 \times 10^6$
18. $17,400$ $1.74 \times 10^4$
19. $500$ $5.0 \times 10^2$
20. $800,000$ $8.0 \times 10^5$
21. $0.00027$ $2.7 \times 10^{-4}$
22. $5300$ $5.3 \times 10^3$
23. $18$ $1.8 \times 10^1$
24. $0.125$ $1.25 \times 10^{-1}$
25. $17,000,000,000$ $1.7 \times 10^{10}$
26. $0.01$ $1.0 \times 10^{-2}$
27. $21,800$ $2.18 \times 10^4$
28. $2,450,000$ $2.45 \times 10^6$
29. $0.0054$ $5.4 \times 10^{-3}$
30. $0.000099$ $9.9 \times 10^{-5}$
31. $8,888,800$ $8.888 \times 10^6$
32. $0.00912$ $9.12 \times 10^{-3}$

Choose the greater number in each pair.

33. $8.8 \times 10^2$, $9.1 \times 10^4$
34. $5.01 \times 10^3$, $5.02 \times 10^1$
35. $6.4 \times 10^6$, 900
36. $1.9 \times 10^8$, 0.02
37. $2.2 \times 10^{-3}$, $2.1 \times 10^{-2}$
38. $8.4 \times 10^6$, 839

Order each set of numbers from least to greatest.

39. $3.6 \times 10^5$, $5.8 \times 10^{-3}$, $2.1 \times 10^3$, $3.5 \times 10^8$
40. $64,000,000$, $6.2 \times 10^6$, $64,000,000$, $6.4 \times 10^9$

Practice

Scientific Notation

Express each number in standard form.

1. $2.4 \times 10^5$ 24,000
2. $9.0 \times 10^3$ 9000
3. $4.385 \times 10^7$ 43,850,000
4. $1.03 \times 10^7$ 103,000,000
5. $3.05 \times 10^5$ 305
6. $5.11 \times 10^8$ 51,100,000
7. $6.000032 \times 10^6$ 6,000,032
8. $1.0 \times 10^{10}$ 10
9. $8.75 \times 10^9$ 875,000
10. $8.49 \times 10^2$ 0.0849
11. $7.1 \times 10^{-4}$ 0.000071
12. $1.0 \times 10^{-3}$ 0.001
13. $4.39 \times 10^{-5}$ 0.000000439
14. $1.25 \times 10^{-4}$ 0.000125

Express each number in scientific notation.

15. $40,000$ $4.0 \times 10^4$
16. $16$ $1.6 \times 10^1$
17. $876,000,000$ $8.76 \times 10^8$
18. $4500$ $4.5 \times 10^3$
19. $151$ $1.51 \times 10^2$
20. $0.0037$ $3.7 \times 10^{-4}$
21. $83,000,000$ $8.3 \times 10^7$
22. $919,100$ $9.191 \times 10^5$
23. $5,000,000,000,000$ $5.0 \times 10^{12}$
24. $0.13$ $1.3 \times 10^{-1}$
25. $0.000007$ $7.0 \times 10^{-7}$
26. $0.0067$ $6.7 \times 10^{-3}$

Order each set of numbers from least to greatest.

27. $7.35 \times 10^3$, $1.7 \times 10^4$, $8.26 \times 10^3$, $9.3 \times 10^3$
28. $0.00048$, $4.37 \times 10^3$, $4.02 \times 10^{-3}$, 0.04
29. $4.37 \times 10^{-4}$, $0.00048$, $4.02 \times 10^{-3}$, 0.04
30. How many drops of water flow over Niagara Falls every minute?

Niagara Falls

For Exercises 29 and 30, use the following information.

Every minute, 840,000,000,000 drops of water flow over Niagara Falls.

29. Write this number in scientific notation.

30. How many drops flow over the falls in a day?

Answers (Lesson 9.5)

Chapter 9

Glencoe Pre-Algebra
9-5 Word Problem Practice

Scientific Notation

1. EARTH SCIENCE Mr. Bell’s class is studying the solar system. The circumference of Earth at the equator is about 24,900 miles. Express this number in scientific notation. \(2.49 \times 10^4\) miles

2. LIGHT SPEED The speed of light is approximately \(6.71 \times 10^8\) miles per hour. Express this number in scientific notation. \(6.71 \times 10^8\) miles per hr

3. EARTH SCIENCE If it takes light 8.3 minutes to reach the Sun from Earth, use the light speed from Exercise 2 to determine the distance from Earth to the Sun. Write your answer in scientific notation. \(9.28 \times 10^7\) mi

4. EARTH SCIENCE The students in Mr. Bell’s class have learned that the mass of Earth is approximately \(5.97 \times 10^{24}\) kilograms. They have also found that mass of an electron is approximately \(9.11 \times 10^{-31}\) kilograms. How many times greater than the mass of an electron is the mass of Earth? \(6.55 \times 10^{27}\) times

5. AIRCRAFT The SR-71 “Blackbird” is one of the world’s fastest airplanes. It is capable of traveling at a cruising speed of Mach 3, or three times the speed of sound. The speed of sound is approximately \(7.5 \times 10^3\) miles per hour. What is Mach 3 in miles per hour? Write your answer in scientific notation. \(2.28 \times 10^4\) mi per hr

d. POPULATION Geographers keep track of how many people live in different areas of the world. They are especially interested in how the populations of certain areas change. The table below shows the population of different regions in 1985 and in 2005.

<table>
<thead>
<tr>
<th>Place</th>
<th>1985</th>
<th>2005</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth</td>
<td>(4.9 \times 10^9)</td>
<td>(6.4 \times 10^9)</td>
</tr>
<tr>
<td>China</td>
<td>(1.1 \times 10^9)</td>
<td>(1.3 \times 10^9)</td>
</tr>
<tr>
<td>India</td>
<td>(7.6 \times 10^8)</td>
<td>(9.1 \times 10^8)</td>
</tr>
<tr>
<td>United States</td>
<td>(2.4 \times 10^9)</td>
<td>(3.0 \times 10^9)</td>
</tr>
</tbody>
</table>

Source: U.S. Census Bureau

a. In 2005, how many times greater than China’s population is the population of the world? \(4.9\) times

b. How many more people inhabited Earth in 2000 than in 1985? \(1.5 \times 10^9\)

c. What was the percentage increase in population in India from 1985 to 2005? Round your answer to the nearest percent. \(45\%\)

d. Was India’s percent increase in population greater than or less than the percent increase of the whole world for the same time period? Explain. Larger. The world’s population increased by 31%.

9-5 Enrichment

Scientific Notation

It is sometimes necessary to multiply and divide very large or very small numbers using scientific notation.

To multiply numbers in scientific notation, use the following rule.

\[ (c \times 10^a)(d \times 10^b) = (c \times d) \times 10^{a+b} \]

Example 1
\[ (3.0 \times 10^4)(5.0 \times 10^3) = (3.0 \times 5.0) \times 10^{4+3} = 15.0 \times 10^7 = 1.5 \times 10^8 \text{ or } 1.5 \times 10^7 \]

To divide numbers in scientific notation, use the following rule.

\[ (c \times 10^a) ÷ (d \times 10^b) = (c ÷ d) \times 10^{a-b} \]

Example 2
\[ (24 \times 10^4) ÷ (1.5 \times 10^3) = (24 ÷ 1.5) \times 10^{4-3} = 16 \times 10^1 = 1.6 \times 10^2 \text{ or } 0.00016 \]

Exercises

Multiply or divide. Express each product or quotient in scientific notation.

1. \(2.7 \times 10^3 \times 3.1 \times 10^3\)
2. \(6.1 \times 10^3 \times 1.3 \times 10^3\)

3. \(5.4 \times 10^3 ÷ (1.8 \times 10^3)\)
4. \(6.9 \times 10^3 ÷ (3.0 \times 10^3)\)

5. \((1.1 \times 10^3) \times (9.9 \times 10^1)\)
6. \((4.0 \times 10^2) ÷ (1.0 \times 10^1)\)

Solve. Write your answers in standard form.

7. ASTRONOMY The distance from Earth to the Moon is about \(2.0 \times 10^5\) miles. The distance from Earth to the Sun is about \(9.3 \times 10^7\) miles. How many times farther is it to the Sun than to the Moon? \(465\)

8. SALARY If each of the \(3.0 \times 10^6\) people employed by Sunny Motors earned \(4.0 \times 10^4\) dollars last year, how much money did the company pay out to its employees? \$1,200,000,000
Powers of Monomials

Power of a Power
You can use the property for finding the product of powers to find a power of a power.

\[(b^m)^n = b^{m \cdot n}\]

The meaning of \((b^m)^n\) should be used as a factor \(n\) times.

Example
Simplify.

\[a. \ (4^3)^5\]

\[= (4^3)^{\cdot 5}\]

\[= 4^{15}\]

\[b. \ (c^3)^7\]

\[= c^{3 \cdot 7}\]

\[= c^{21}\]

Exercises
Simplify.

1. \((7^2)^3\)
2. \((12^2)^3\)
3. \((8^2)^3\)
4. \((22^2)^3\)

1. \(7^{12}\)
2. \(12^{21}\)
3. \(8^{25}\)
4. \(22^6\)

5. \((a^3)^5\)
6. \((y^5)^2\)
7. \((b^2)^3\)
8. \((r^3)^1\)

9. \((4^3)^5\)
10. \((-8)^3\)
11. \((5^3)^6\)
12. \((-10^3)^2\)

13. \((t^2)^3\)
14. \((-3^2)^3\)
15. \((c^3)^4\)
16. \((d^2)^3\)

13. \(4^{15}\)
14. \((-3^2)^3\)
15. \((c^3)^4\)
16. \((d^2)^3\)

Power of a Product
To find the power of a product, find the power of each factor and multiply.

\[(a^m)(b^n) = a^m \cdot b^n\]

Example
Simplify.

\[a. \ (7x^4)^2\]

\[= 7^2 \cdot (x^4)^2\]

\[= 49 \cdot x^{8}\]

\[b. \ (3xy^2)^3\]

\[= 3^3 \cdot (xy^2)^3\]

\[= 27 \cdot x^3 \cdot y^6\]

Exercises
Simplify.

1. \((6x^3)^2\)
2. \((5y^4)^2\)
3. \((12y^3)^2\)
4. \((-8x^2)^2\)

1. \(216x^{12}\)
2. \(625y^{12}\)
3. \(144y^{14}\)
4. \(-512x^8\)

5. \((11x^2)^2\)
6. \((7x^3)^2\)
7. \((4y^2)^3\)
8. \((2x^2)^4\)

9. \((6p^2q^3)^2\)
10. \((-9m^3n^2)^2\)
11. \((10pq^5)^4\)
12. \((5d^5e^3)^4\)

13. \((-4y^2)^2\)
14. \((3x^3y)^3\)
15. \((6a^2b^3)^2\)
16. \((-10c^2d^3)^2\)

13. \(256x^{10}y^{12}\)
14. \(81x^{10}y^{12}\)
15. \(512a^{12}b^{12}\)
16. \(10,000x^{10}y^{12}\)
9-6 Skills Practice

Powers of Monomials

Simplify.

1. \((7^5)^2\)  
2. \((12^3)^4\)  
3. \((-15)^{14}\) or \(15^{24}\)

4. \((a^2)^3\)  
5. \((6^3)^5\)  
6. \((-f)^2\) or \(f^{14}\)

7. \((k^4)^2\)  
8. \((a^3)^4\)  
9. \((10^9)^8\)

10. \((42^9)^4\)  
11. \((m^3)^2\)  
12. \((p^4)^7\)

13. \((90^5)^9\)  
14. \((-3^3)^4\)  
15. \((-b)^{12}\) or \(b^{12}\)

16. \((8^3)^8\)  
17. \((12^2)^3\)  
18. \((-7c)^{18}\)

19. \((11^3)^5\)  
20. \((3p^4)^5\)  
21. \((-5q)^{15}\)

22. \((40^2)^3\)  
23. \((-100^3)^2\)  
24. \((9r^2)^3\)

25. \((-512x^2y^3)^3\)  
26. \((2g^3h^2)^7\)  
27. \((-3a^5b^2)^4\)

28. \((20^3u^2)^3\)  
29. \((40a^2b^3)^3\)  
30. \((13p^2q)^3\)

8,000x^{19}y^{31}z^4  
1,600x^{19}y^{44}z^4  
169f^{10}g^4
**Powers of Monomials**

1. **AREA** Haley's math teacher drew the square shown below. Find the area of the square.

   \[ \text{Area} = d^2 \]

2. **PYRAMIDS** The Great Pyramid in Giza has a square base that measures 755 \( \times \) 10\(^{3} \) feet. Find the area of the base of the pyramid. 570025 \( \times \) 10\(^{6} \) square feet

3. **ATOMS** Electrons are found in the nucleus of atoms. The radius of an electron is 2.8179 \( \times \) 10\(^{-9} \) meters. Use the formula \( V = \frac{4}{3} \pi r^3 \) to find the volume of an electron. Round to the nearest thousandth. 3.324 \( \times \) 10\(^{-6} \) cubic meter

4. **VOLUME** What is the volume of a cube with sides that measure 5\( \times \)10\(^{-3} \)? (Hint: \( V = s^3 \)) 125\( \times \)10\(^{-9} \)

5. **MINERALS** A grain of salt is cubic in shape. Each side measures about 5 \( \times \) 10\(^{-4} \) mm in length. Use the formula \( S^3 = 6s^3 \) to find the surface area of a grain of salt. 1.5 square millimeters

6. **CELESTIAL BODIES** The table below lists the radii of different celestial bodies.

<table>
<thead>
<tr>
<th>Celestial Body</th>
<th>Radius (mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moon</td>
<td>1.08 ( \times ) 10(^{2} )</td>
</tr>
<tr>
<td>Earth</td>
<td>3.959 ( \times ) 10(^{2} )</td>
</tr>
<tr>
<td>Sun</td>
<td>4.30 ( \times ) 10(^{2} )</td>
</tr>
</tbody>
</table>

a. Use the formula \( V = \frac{4}{3} \pi r^3 \) to find the volume of the Moon. Round to the nearest million. 5.274 \( \times \) 10\(^{7} \) cubic miles

b. Use the formula \( A = 4 \cdot 3.14 \cdot r^2 \) to find the surface area of the Sun. Round to the nearest billion. 2.344 \( \times \) 10\(^{8} \) square miles

c. Use the formula \( A = 3.14 \cdot r^2 \) to find the area of the cross section of Earth at the equator. Round to the nearest million. 4.9 \( \times \) 10\(^{7} \)

**Fractional Exponents**

A radical sign (\( \sqrt[\cdot]{\cdot} \)) indicates the square root of a number. The square root is one of the number's two equal factors. You have seen that both positive and negative integers can be exponents. A fraction can also be an exponent. Fractional exponents can be rewritten in radical form:

\[
25^{\frac{1}{2}} = \sqrt{25} = 5, \text{ since } 5 \cdot 5 = 25
\]

If \( x > 0 \) and \( p \) and \( q \) are integers and \( q > 0 \), then \( x^\frac{p}{q} = (\sqrt[q]{x})^p = \sqrt[q]{x^p} \).

The denominator of the fractional exponent indicates the root:

\[
1,000^{\frac{1}{3}} = \sqrt[3]{1,000} = 10, \text{ since } 10^3 = 1,000.
\]

The numerator of the fractional exponent indicates the power to which to raise the root:

\[
10,000^{\frac{1}{4}} = (\sqrt[4]{10,000})^3 = 10, \text{ since } 10^4 = 10,000.
\]

### Exercises

#### Simplify each expression.

**Example**

\[
16^{\frac{1}{2}} = \sqrt{16} = 4 \quad 27^{\frac{1}{3}} = (\sqrt[3]{27})^2 = 9
\]

The cube root of 27 is 3, since 3\(^3\) = 27. Simplify.

a. \( 16^{\frac{1}{2}} \)

b. \( 27^{\frac{1}{3}} \)

c. \( 64^{\frac{1}{2}} \)

d. \( 125^{\frac{1}{3}} \)

e. \( 8^{\frac{1}{3}} \)

#### Simplify each expression.

1. \( 81^{\frac{1}{2}} \)

2. \( 9^{\frac{1}{3}} \)

3. \( 49^{\frac{1}{2}} \)

4. \( 36^{\frac{1}{2}} \)

5. \( 64^{\frac{1}{3}} \)

6. \( 125^{\frac{1}{3}} \)

7. \( 216^{\frac{1}{3}} \)

8. \( 64^{\frac{1}{2}} \)

9. \( 8^{\frac{1}{3}} \)

10. \( 512^{\frac{1}{3}} \)

11. \( 216^{\frac{1}{3}} \)

12. \( 125^{\frac{1}{2}} \)

13. \( 2^{\frac{1}{3}} \)

14. \( 8^{\frac{1}{2}} \)

15. \( 16^{\frac{1}{2}} \)

16. \( 100^{\frac{1}{2}} \)

17. \( 1,000^{\frac{1}{3}} \)

18. \( 81^{\frac{1}{2}} \)
Chapter 9

9-7 Study Guide and Intervention

Linear and Nonlinear Functions

Graphs of Nonlinear Functions

Linear functions are relations with a constant rate of change. Graphs of linear functions are straight lines. Nonlinear functions do not have a constant rate of change. Graphs of nonlinear functions are not straight lines.

Example

Determine whether each graph represents a linear or nonlinear function. Explain.

a.

This graph is a curve, not a straight line. So, it represents a nonlinear function.

b.

This graph is a line. So, it represents a linear function.

Exercises

Determine whether each graph represents a linear or nonlinear function. Explain.

1. 

This graph is a curve, not a straight line. So, it represents a nonlinear function.

2. 

This graph is a line. So, it represents a linear function.

3. 

This graph is a curve, not a straight line. So, it represents a nonlinear function.

4. 

This graph is a line. So, it represents a linear function.

Table of Contents

Lesson 9-7

Linear and Nonlinear Functions

Equations and Tables

Linear functions have constant rates of change. Their graphs are straight lines and their equations can be written in the form $y = mx + b$. Nonlinear functions do not have constant rates of change and their graphs are not straight lines.

Example 1

Determine whether each equation represents a linear or nonlinear function. Explain.

a. $y = 9$

This is linear because it can be written as $y = 0x + 9$.

b. $y = x^2 + 4$

This is nonlinear because the exponent of $x$ is not 1, so the equation cannot be written in the form $y = mx + b$.

Tables can represent functions. A nonlinear function does not increase or decrease at a constant rate.

Example 2

Determine whether each table represents a linear or nonlinear function. Explain.

a. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-7</td>
</tr>
<tr>
<td>+2</td>
<td>+8</td>
</tr>
</tbody>
</table>

As $x$ increases by 2, $y$ increases by 8. The rate of change is constant, so this is a linear function.

b. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>10</td>
<td>75</td>
</tr>
</tbody>
</table>

As $x$ increases by 5, $y$ decreases by a greater amount each time. The rate of change is not constant, so this is a nonlinear function.

Exercises

Determine whether each equation or table represents a linear or nonlinear function. Explain.

1. $x + 3y = 9$ Linear; equation can be written as $y = -\frac{1}{3}x + 3$.

2. $y = \frac{5}{x}$ Nonlinear; equation cannot be written in the form $y = mx + b$.

3. $y = 6(x + 1)$ Nonlinear; equation cannot be written in the form $y = mx + b$. written as $y = -5x + 9$.

4. $y = 9 - 5x$ Linear; equation can be written in the form $y = mx + b$.

5. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>24</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>-6</td>
</tr>
</tbody>
</table>

Linear; rate of change is constant.

6. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>4</td>
<td>54</td>
</tr>
</tbody>
</table>

Nonlinear; rate of change is not constant.
9-7 Skills Practice

Linear and Nonlinear Functions

Determine whether each graph, equation, or table represents a linear or nonlinear function. Explain.

1. Nonlinear; the graph is a curve.

2. Linear; the graph is a straight line.

3. Nonlinear; the graph is a curve.

4. $y = \frac{x}{2} + 1$

5. $y = -\frac{x}{2} + 10$

6. $y = 8x$

7. $y = 6$

8. $2x - y = 5$

9. $y = x^2 + 4$

10. $y + 4x^2 - 1 = 0$

11. $2y - 8x + 11 = 0$

12. $y = \sqrt[x]{x} - 2$

13. $x | y$
   1 | 8
   2 | 5
   3 | 2
   4 | -1

14. $x | y$
   6 | 1
   12 | 3
   18 | 6
   24 | 10

15. $x | y$
   20 | -4
   15 | -2
   10 | 0
   5 | 2

16. Nonlinear; rate of change is not constant.

17. Nonlinear; rate of change is not constant.

18. Linear; rate of change is constant.

19. Linear; rate of change is constant.

20. Nonlinear; rate of change is not constant.

Geometry

The graph shows how the area of a square increases as the perimeter increases. Is this relationship linear or nonlinear? Explain.

Nonlinear; the graph is not a straight line.
## Word Problem Practice

### Linear and Nonlinear Functions

1. **TEMPERATURE** In the United States, temperature is most often measured in degrees Fahrenheit. Temperature is measured in degrees Celsius in the metric system. The formula used to convert between these two units of measure is \( F = \frac{9}{5}C + 32 \) where \( F \) represents degrees Fahrenheit and \( C \) represents degrees Celsius. Does this equation represent a **linear** or non-linear function?

2. **COMPUTER GAMES** Suppose the function \(-0.005d^2 + 0.13d = h\) is used to simulate the path of a golf ball that is hit off a tee in a computer game. Does this equation represent a **linear** or non-linear function?

3. **GASOLINE** The table below shows gasoline prices in Springfield during a one-month period. Is the change in gas price a linear function? Explain.

### Polynomial Functions

#### 4. Football Punts

The function \( h = -16t^2 + 90t + 1.5 \) represents the height \( h \) of the football, in feet, after \( t \) seconds when a punter kicks the ball with an upward velocity of 90 feet per second and his foot meets the ball 1.5 feet off the ground. Is this a linear function of time? Explain.

**Nonlinear.** The height of the football will vary, so the change is not constant.

#### 5. Flight Research

The equation \( h = -16t^2 + 60t + 4482 \) represents the height, \( h \), in feet, of a pilot over time, \( t \), in seconds, after he or she has ejected from a jet and falls to Earth with the aid of a parachute. A pilot is flying at an altitude of approximately 10,000 feet and is forced to eject from the jet. The equation \( h = 10,000 \) represents an altitude of 10,000 feet.

### Gasoline Prices

The table below shows gasoline prices in Springfield during a one-month period. Is the change in gas price a linear function? Explain.

**Nonlinear;** the amount of change in price each day is not constant.

<table>
<thead>
<tr>
<th>Day of the Month</th>
<th>Price per Gallon</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2.57</td>
</tr>
<tr>
<td>4</td>
<td>$2.72</td>
</tr>
<tr>
<td>7</td>
<td>$2.72</td>
</tr>
<tr>
<td>10</td>
<td>$2.88</td>
</tr>
<tr>
<td>13</td>
<td>$2.88</td>
</tr>
<tr>
<td>16</td>
<td>$2.84</td>
</tr>
<tr>
<td>19</td>
<td>$2.76</td>
</tr>
<tr>
<td>21</td>
<td>$2.72</td>
</tr>
<tr>
<td>24</td>
<td>$2.64</td>
</tr>
<tr>
<td>27</td>
<td>$2.60</td>
</tr>
<tr>
<td>30</td>
<td>$2.52</td>
</tr>
</tbody>
</table>

**a.** Which equation is a linear function?

\[ h = 10,000 \]

**b.** Explain why the other equation is a non-linear function. **Sample answer:** The other equation is of the form \( y = ax^2 + bx + c \), which is a quadratic equation.

### Gasoline Prices

<table>
<thead>
<tr>
<th>Day of the Month</th>
<th>Price per Gallon</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
<td>7</td>
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</tr>
<tr>
<td>10</td>
<td>$2.88</td>
</tr>
<tr>
<td>13</td>
<td>$2.88</td>
</tr>
<tr>
<td>16</td>
<td>$2.84</td>
</tr>
<tr>
<td>19</td>
<td>$2.76</td>
</tr>
<tr>
<td>21</td>
<td>$2.72</td>
</tr>
<tr>
<td>24</td>
<td>$2.64</td>
</tr>
<tr>
<td>27</td>
<td>$2.60</td>
</tr>
<tr>
<td>30</td>
<td>$2.52</td>
</tr>
</tbody>
</table>

**Answers (Lesson 9-7)**

#### 9-7 Enrichment

**David R. Hedgley**

African-American mathematician David R. Hedgley, Jr. (1937–) solved one of the most difficult problems in the field of computer graphics—how to program a computer to show any three-dimensional object from a given viewpoint just as the eye would see it. Hedgley's solution helped researchers in aircraft experimentation. Hedgley received an M.S. in Mathematics from California State University in 1970 and a Ph.D. in Computer Science from Somerset University in England in 1988. Hedgley has received numerous national achievement awards.

Polynomials in three variables are needed to describe some three-dimensional objects. Each variable represents one of the three dimensions: height, width, and depth.

\[ P_1 = x^2 + y^2 + z^2 + 10x + 4y + 3z - 19 \]

\[ P_2 = 2x^2 + 2y^2 + 2z^2 - 2x - 3y + 5z - 2 \]

1. **Add the polynomials** \( P_1 \) **and** \( P_2 \).

\[ 3x^2 + 3y^2 + 3z^2 + 8x + y + 7z - 21 \]

2. **Subtract the polynomials** \( P_1 \) **from** \( P_2 \).

\[ x^2 + y^2 + z^2 - 12x - 7y + 3z + 17 \]

If the polynomials above were each set equal to zero, they would form equations describing two different spheres in three-dimensional space, or 3-space. The coordinate plane you studied in Chapter 2 represents two-space. You described most lines in that plane by an equation in two variables. Each point on a line could be written as an ordered pair of numbers \((x, y)\). Each point on any figure in 3-space can be written as an ordered triple of numbers \((x, y, z)\).

3. **What are the values of** \( x, y, \) **and** \( z \) **for point** \( A \) **in the diagram?**

\[ x = 14, \ y = 4, \ z = 8 \]

4. **Give the ordered triple representing each of the points** \( B \) **through** \( G \) **in the diagram.**

\[ B(14, 0, 8), \ C(0, 4, 8), \ D(0, 0, 8), \ E(14, 4, 0), \ F(14, 0, 0), \ G(0, 0, 0) \]
Graph Quadratic Functions  Functions which can be described by an equation of the form \( y = ax^2 + bx + c \), where \( a \neq 0 \), are called quadratic functions. The graph of a quadratic equation takes the form shown to the right, which is called a parabola.

Just as with linear functions, you can graph quadratic functions by making a table of values.

**Example** Graph \( y = x^2 - 3 \).

Make a table of values, plot the ordered pairs, and connect the points with a curve.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = x^2 - 3 )</th>
<th>( x, y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>(-2^2 - 3 = -1)</td>
<td>(-2, -1)</td>
</tr>
<tr>
<td>-1</td>
<td>((-1)^2 - 3 = -2)</td>
<td>(-1, -2)</td>
</tr>
<tr>
<td>0</td>
<td>((0)^2 - 3 = -3)</td>
<td>(0, -3)</td>
</tr>
<tr>
<td>1</td>
<td>((1)^2 - 3 = -2)</td>
<td>(1, -2)</td>
</tr>
<tr>
<td>2</td>
<td>((2)^2 - 3 = 1)</td>
<td>(2, 1)</td>
</tr>
</tbody>
</table>

**Exercises**

Graph each function.

1. \( y = x^2 + 2 \)
2. \( y = -x^2 + 2 \)
3. \( y = x^2 - 2 \)
4. \( y = 3x^2 - 1 \)
5. \( y = \frac{1}{4}x^2 \)
6. \( y = -2x^2 + 3 \)

**Example**

**MAPS** The principal of Smithville Elementary wants to paint a map of the U.S. on the cafeteria wall. Before the map can be painted, the rectangular space where the map will go must be painted white. The height of the rectangle will be \( \frac{2}{3} \) the width.

a. Graph the equation that gives the area for the rectangle for different lengths and widths. What is the area of the rectangle with a width of 10 feet? What is the length?

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = \frac{2}{3}x^2 )</th>
<th>( x, y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>( \frac{3}{2}(4)^2 )</td>
<td>(4, 9.6)</td>
</tr>
<tr>
<td>6</td>
<td>( \frac{3}{2}(6)^2 )</td>
<td>(6, 21.6)</td>
</tr>
<tr>
<td>8</td>
<td>( \frac{3}{2}(8)^2 )</td>
<td>(8, 38.4)</td>
</tr>
<tr>
<td>10</td>
<td>( \frac{3}{2}(10)^2 )</td>
<td>(10, 60)</td>
</tr>
<tr>
<td>12</td>
<td>( \frac{3}{2}(12)^2 )</td>
<td>(12, 86.4)</td>
</tr>
</tbody>
</table>

The area of the rectangle when the width is 10 feet is 60 square feet. The length is 6 feet.

b. What values of the domain and range are unreasonable? Explain.

Unreasonable values of the domain and range would be any negative numbers because neither the length nor the width can be negative.

**Exercise**

1. **GRAVITY** An object is dropped from a height of 300 feet. The equation that gives the object's height in feet \( h \) as a function of time \( t \) is \( h = -16t^2 + 300 \). Graph this equation and interpret your graph. What was the height of the object after 4 seconds?

The height of the object at 0 s when it was dropped was 300 feet. The object reached a height of 0 ft (or it landed on the ground) at about 4.3 s. The height of the object at 4 s was 44 ft.
Graph each function.

1. \( y = 5x^2 \)
2. \( y = -x^2 \)
3. \( y = -5x^2 \)
4. \( y = x^2 - 1 \)
5. \( y = x^2 + 4 \)
6. \( y = -2x^2 + 2 \)
7. \( y = x^2 - 4 \)
8. \( y = 2x^2 - 2 \)

5. WINDOWS A window maker has 25 feet of wire to frame a window. One side of the window is \( x \) feet and the other side is \( 9 - x \) feet.
   a. Write an equation to represent the area \( A \) of the window.
      \[ A = -x^2 + 9x \]
   b. Graph the equation you wrote in part a.
   c. If the area of the window is 18 square feet, what are the two possible values of \( x \)?
      \( x = 3 \) and \( x = 6 \)
9-8 Word Problem Practice

Quadratic Functions

1. RACING Between the ages of 8 and 16, Houston native Erica Enders won 37 junior dragster races. The distance her car travels down the drag strip can be expressed by the equation \( d = \frac{1}{2}at^2 \), where \( a \) is the rate of acceleration and \( t \) is time. Suppose her car accelerates at a rate of 49.5 feet per second. Find the number of feet her car traveled after 7 seconds.

2. PHYSICS The top of the Leaning Tower of Pisa is 185 feet above the ground. Suppose an object is dropped from the top of the Leaning Tower of Pisa. The height \( h \) in feet of the object, after \( t \) seconds, is represented by the equation \( h = 185 - 16t^2 \). How far from the ground is it after 3 seconds?

3. VISTAS The Texas State Capitol building is 311 feet tall. The formula \( d = \frac{3}{4}a \) represents the number of miles \( d \) that a person can see from an altitude of \( a \) feet. Graph the function and use it to estimate how far you could see from the top of the Texas State Capitol.

4. FIREWORKS The largest annual pyrotechnic display in North America is Thunder over Louisville held to kick off the Kentucky Derby Festival. The table shows the larger shell sizes and their corresponding velocities.

<table>
<thead>
<tr>
<th>Shell Size (in.)</th>
<th>Initial Velocity (ft/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>235</td>
</tr>
<tr>
<td>10</td>
<td>263</td>
</tr>
<tr>
<td>12</td>
<td>287.5</td>
</tr>
<tr>
<td>24</td>
<td>393</td>
</tr>
<tr>
<td>36</td>
<td>481</td>
</tr>
</tbody>
</table>

Source: ThinkQuest

For each shell size:
- a. The equation \( h = -16t^2 + 235t + 3 \) represents the height \( h \) in feet of an 8-inch shell \( t \) seconds after it is launched from 3 feet with an initial velocity of 235 feet per second. Graph the equation.
- b. How high is the shell after 5 seconds?

Shell Size 8: 778 feet

Sample answer: about 20 miles (exact answer 21.6 miles)

9-8 Enrichment

Translating Quadratic Graphs

When a figure is moved to a new position without undergoing any rotation, then the figure is said to have been translated to the new position.

The graph of a quadratic equation in the form \( y = (x - b)^2 + c \) is a translation of the graph of \( y = x^2 \).

Start with a graph of \( y = x^2 \).

Slide to the right 4 units.

Then slide up 3 units.

The following equations are in the form \( y = x^2 + c \). Graph each equation.
1. \( y = x^2 + 1 \)
2. \( y = x^2 + 2 \)
3. \( y = x^2 - 2 \)

The following equations are in the form \( y = (x + b)^2 \). Graph each equation.
4. \( y = (x + 1)^2 \)
5. \( y = (x - 3)^2 \)
6. \( y = (x + 2)^2 \)
Chapter 9

9-8 Spreadsheet Activity

Families of Quadratic Graphs

A family of graphs is a group of graphs that have at least one characteristic in common. You can use a spreadsheet to study the characteristics of families of quadratic graphs.

Example

Graph the quadratics \(y = x^2\), \(y = 2x^2\), and \(y = 4x^2\). What are the similarities and differences among the graphs?

Step 1

Use Column A for the values of \(x\) and Columns B, C, and D for the values of \(y\). Exponents are entered using the \(^\text{^}\) symbol. For example, cell B2 contains the formula \(A2^2\).

Step 2

To create a graph from the data, select the data in Columns A, B, C, and D and choose Chart from the Insert menu. Select an XY (Scatter) chart with a smooth line to show the graphs.

The graphs of all three functions pass through the point at \((0, 0)\).

The graph of \(y = x^2\) is wider than the graph of \(y = 2x^2\). The graph of \(y = 2x^2\) is wider than the graph of \(y = 4x^2\).

Exercises

1. Make a conjecture about the graph of \(y = \frac{1}{2}x^2\) as compared to the graphs above. Use the spreadsheet to graph \(y = \frac{1}{2}x^2\) and verify your conjecture. It passes through \((0, 0)\) and is wider than the graph of \(y = x^2\).

2. Graph the quadratics \(y = x^2\), \(y = x^2 + 2\), and \(y = x^2 - 3\). What are the similarities and differences among the graphs? See students graphs. The graphs are all the same shape, but the graph of \(y = x^2\) passes through \((0, 0)\), \(y = x^2 + 2\) passes through \((0, 2)\), and \(y = x^2 - 3\) passes through \((0, -3)\).

9-9 Study Guide and Intervention

Cubic and Exponential Functions

Cubic Functions

Functions which can be described by an equation of the form \(y = ax^3 + bx^2 + cx + d\), where \(a \neq 0\), are called cubic functions. The graph of a cubic equation takes the form shown to the right.

Just as with linear and quadratic functions, you can graph cubic functions by making a table of values.

Example

Graph \(y = 2x^3 - 1\).

Make a table of values, plot the ordered pairs, and connect the points with a curve.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y = 2x^3 - 1)</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>(-2 - 1) - 1 = -3)</td>
<td>((-1, -3))</td>
</tr>
<tr>
<td>0</td>
<td>(2(0)^3 - 1 = -1)</td>
<td>((0, -1))</td>
</tr>
<tr>
<td>1</td>
<td>(2(1)^3 - 1 = 1)</td>
<td>((1, 1))</td>
</tr>
<tr>
<td>1.2</td>
<td>(2(1.2)^3 - 1 = 2.5)</td>
<td>((1.2, 2.5))</td>
</tr>
</tbody>
</table>

Exercises

Graph each function.

1. \(y = x^3 + 2\)
2. \(y = -x^3 + 2\)
3. \(y = x^3 - 2\)
4. \(y = 2x^3\)
5. \(y = -2x^3 + 2\)
6. \(y = \frac{1}{6}x^3 - 1\)
Cubic and Exponential Functions

Exponential Functions In linear, quadratic, and cubic functions, the variable is the base. Exponential functions are functions in which the variable is the exponent rather than the base. An exponential function is a function that can be described by an equation of the form \( y = a^x + c \), where \( a \neq 0 \) and \( a \neq 1 \).

Example Graph \( y = 3^x - 6 \).

First, make a table of ordered pairs. Then graph the ordered pairs.

\[
\begin{array}{|c|c|c|}
\hline
x & y = 3^x - 6 & (x, y) \\
\hline
-2 & y = 3^{-2} - 6 \approx -5.9 & (-2, -5.9) \\
-1 & y = 3^{-1} - 6 \approx -5.7 & (-1, -5.7) \\
0 & y = 3^0 - 6 = -5 & (0, -5) \\
1 & y = 3^1 - 6 = -3 & (1, -3) \\
2 & y = 3^2 - 6 \approx 3 & (2, 3) \\
\hline
\end{array}
\]

Exercises

Graph each function.

1. \( y = 2^x \)
2. \( y = 3^x + 2 \)
3. \( y = 2^x - 1 \)
4. \( y = 2^x + 1 \)
5. \( y = 3^x - 3 \)
6. \( y = 4^x - 6 \)
7. \( y = 2^x - 4 \)
8. \( y = 3^x - 7 \)
9-9 **Practice**

**Cubic and Exponential Functions**

Graph each function.

1. \( y = 0.4x^3 \)
2. \( y = -2x^3 - 1 \)

3. \( y = x^3 + 0.5 \)
4. \( y = \frac{1}{2}x^3 \)

5. \( y = 3^x + 0.75 \)
6. \( y = 3^x - 4 \)

7. **E-MAIL** Mike forwarded an e-mail to 5 friends. Each of those 5 friends forwarded it to 5 of their friends. Each of those friends forwarded it to five friends and so on. The function \( N = 5^x \) represents the total number of e-mails forwarded, where \( x \) is the stage of the e-mails. Graph the function. In what stage will the number of e-mails forwarded be at least 625?

---

9-9 **Word Problem Practice**

**Cubic and Exponential Functions**

1. **VOLUME** The equation \( y = \frac{3}{2} \pi x^3 \) represents the volume of a sphere. Graph the equation in the first quadrant.

2. **GEOMETRY**

   Write the function for the volume of a cone as a function of a radius \( r \) units if the height equals the radius. Then graph the function.
   \( V = \frac{1}{3} \pi r^2 \)

3. **MONEY** Sam’s teacher wrote the problem shown below on the board.

   You have a choice of receiving 1 million dollars all at once, or, to receive a penny one day, then double this number of pennies the next day, then double that number on the third day and so on, for 30 days. Which is the better deal?

   The function representing the number of pennies is \( y = 2^x - 1 \), where \( x \) is the number of days. Graph the function. Is it better to take the million dollars or the pennies? Explain your answer.

   It is better to take the pennies. On day 30 you will have a total of 536,870,912 pennies, which is $5,368,709.12.

4. **MARATHONS** Helen runs 5 miles every other day. She wants to increase the number of miles she runs to train for a marathon. She plans to increase the mileage according to the function \( y = 5(1.1)^x \), where \( x \) represents the number of times she goes running.

   Graph the function. On which day will Helen first run 26 miles? day 18

5. **POPULATION GROWTH** The population of the town of Carlyll is growing at a rate of 5.5% per year. There are currently 1,050 people in the town.

   a. Graph the function \( y = 1,050(1.055)^x \), where \( x \) represents the number of years.

   b. Identify the \( y \)-intercept and explain what it means. The \( y \)-intercept is 1,050, the number of people currently living in Carlyll.

   c. Estimate the number of people who will live in Carlyll in 12 years.

   d. Estimate the number of people who lived in Carlyll 5 years ago.

   about 800
### 9-9 Enrichment

**Writing Exponential Growth and Decay Equations**

Exponential functions can be used to represent real-world situations involving both exponential growth and exponential decay. To determine whether an equation in the form $y = a \cdot b^x$ represents exponential growth or decay, look at the value of $b$. If the value of $b$ is greater than 1, the equation represents exponential growth. If the value of $b$ is less than 1, the equation represents exponential decay.

**Example**

**Words:** The population of a town with 1,000 residents is increasing at a rate of 8% per year.

**Symbols:** $y = 1000(1.08)^x$

**Graph:**

This shows exponential growth.

**Words:** The population of a town with 1,000 residents is decreasing at a rate of 8% per year.

**Symbols:** $y = 1000(0.92)^x$

**Graph:**

This shows exponential decay.

A simple formula for writing exponential growth and decay equations is shown below.

<table>
<thead>
<tr>
<th>Growth</th>
<th>Decay</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = a(1 + r)^x$</td>
<td>$y = a(1 - r)^x$</td>
</tr>
</tbody>
</table>

*Where:*

- $a =$ initial amount before growth/decay
- $r =$ rate of growth/decay
- $x =$ number of time intervals

**Exercises**

Write an equation to represent each situation.

1. Simple interest on $600 at 3%. $y = 600(1.03)^x$
2. There are 200 competitors in a tournament. One-quarter of the competitors are eliminated at the end of each round. $y = 200(0.75)^x$
3. The median home price of $350,000 increases at a rate of 4% a year. $y = 350,000(1.04)^x$
4. A cable company had 10,000 subscribers. The number of subscribers decreased by 2.2% each month. $y = 10,000(0.978)^x$
5. A barometric pressure reading of 30 decreases by about 12% per kilometer above sea level. $y = 30(0.88)^x$
Chapter 9 Assessment Answer Key

Quiz 1 (Lessons 9-1 through 9-3) Page 64

1. \[ x^3 \]
2. \[ 9^5 \]
3. \[ 192 \]
4. \[ 72 \]
5. \[ 3^3 \]
6. \[ 3^2 \cdot 7 \]
7. \[ 3 \cdot 7 \cdot b \]
8. \[ 2 \cdot 3 \cdot 5 \cdot x \cdot x \cdot y \]
9. \[ -3r^9 \]
10. \[ a^{12} \]

Quiz 2 (Lessons 9-4 and 9-5) Page 64

1. \[ \frac{1}{7^5} \]
2. \[ \frac{1}{y^3} \]
3. \[ \frac{1}{81} \]
4. \[ \frac{1}{216} \]
5. \[ \frac{1}{4} \]
6. \[ 1.602 \times 10^{-7} \]
7. \[ 2.0 \times 10^8 \]
8. \[ 1.5 \times 10^1 \]
9. \[ 4.62 \times 10^{-3} \]
10. \[ 6.25 \times 10^4 \]

Sample answer: The graph of \( y = 2x^2 \) is wider than the graph of \( y = x^2 \). Both graphs have the same \( x \)-and \( y \)-intercepts.

Quiz 3 (Lessons 9-6 and 9-7) Page 65

1. \[ 8x^{12} \]
2. \[ \frac{1}{-27y^6} \]
3. Nonlinear; the graph is a curve.
4. Linear; rate of change is constant.
5. \[ B \]

Quiz 4 (Lessons 9-8 and 9-9) Page 65

1. \[ \frac{2}{3} \cdot \frac{2}{3} \]
2. \[ \frac{2 \cdot 3 \cdot 11}{2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x} \]
3. \[ x \cdot y \cdot y \]
4. \[ n^4 \]
5. \[ 7^{-2} \]

Mid-Chapter Test Page 66

1. \[ C \]
2. \[ G \]
3. \[ D \]
4. \[ G \]
5. \[ B \]
1. g
2. i
3. c
4. b
5. d
6. f
7. e
8. h
9. a

10. Sample answer: A number expressed in scientific notation is written as the product of a factor and a power of 10. The factor must be greater than or equal to 1 and less than 10.

11. Sample answer: In a power, the base is the number that is multiplied.

12. Sample answer: A nonlinear function is a function that does not have a constant rate of change. Its graph is not a straight line.

13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____

B: 8 people
<table>
<thead>
<tr>
<th>Form 2A</th>
<th>Page 70</th>
<th>Form 2B</th>
<th>Page 72</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. <strong>D</strong></td>
<td>12. <strong>J</strong></td>
<td>1. <strong>D</strong></td>
<td>12. <strong>F</strong></td>
</tr>
<tr>
<td>2. <strong>H</strong></td>
<td>13. <strong>B</strong></td>
<td>2. <strong>H</strong></td>
<td>13. <strong>D</strong></td>
</tr>
<tr>
<td>3. <strong>D</strong></td>
<td>14. <strong>F</strong></td>
<td>3. <strong>B</strong></td>
<td>14. <strong>F</strong></td>
</tr>
<tr>
<td>4. <strong>G</strong></td>
<td>15. <strong>D</strong></td>
<td>4. <strong>J</strong></td>
<td>15. <strong>C</strong></td>
</tr>
<tr>
<td>5. <strong>A</strong></td>
<td>16. <strong>H</strong></td>
<td>5. <strong>B</strong></td>
<td>17. <strong>A</strong></td>
</tr>
<tr>
<td>7. <strong>C</strong></td>
<td>18. <strong>J</strong></td>
<td>7. <strong>A</strong></td>
<td>19. <strong>C</strong></td>
</tr>
<tr>
<td>10. <strong>H</strong></td>
<td></td>
<td>10. <strong>J</strong></td>
<td></td>
</tr>
<tr>
<td>11. <strong>B</strong></td>
<td></td>
<td>11. <strong>C</strong></td>
<td></td>
</tr>
</tbody>
</table>
Chapter 9 Assessment Answer Key

Form 2C
Page 74

1. \(2^5\)

2. \(56\)

3. \(2 \cdot 2 \cdot 2 \cdot 11\)

4. \(2 \cdot 3 \cdot 7 \cdot a \cdot a \cdot a \cdot x\)

5. \(m^7\)

6. \(55x^3y^2\)

7. \((-4)^5\)

8. \(b^{10}\)

9. \(3^{-9}\)

10. \(\frac{1}{125}\)

11. \(0.000509\)

12. \(1.5 \times 10^9\)

13. Mercury

Page 75

14. \(d^{24}\)

15. \(27v^{12}\)

16. Nonlinear; the graph is not a straight line.

17. Linear; rate of change is constant.

18. [Graph of a nonlinear function]

19. [Graph of a linear function]

20. [Graph of a linear function]

B: \(2^2 \cdot 3^3 \cdot 11\)
Chapter 9 Assessment Answer Key

Form 2D
Page 76

1. \[ 3^6 \]

2. \[ 405 \]

3. \[ 2 \cdot 2 \cdot 2 \cdot 17 \]

4. \[ 2 \cdot p \cdot p \cdot x \]

5. \[ d^5 \]

6. \[ 36x^2y^2 \]

7. \[ (-3)^3 \]

8. \[ a^{15} \]

9. \[ 10^{-3} \]

10. \[ \frac{1}{16} \]

11. \[ 0.000468 \]

12. \[ 6.35 \times 10^{11} \]

13. human being

Page 77

14. \[ z^{48} \]

15. \[ 32u^{25} \]

16. Linear; the graph is a straight line.

17. Nonlinear; rate of change is not constant.

18.

19.

20.

B: \[ 2^4 \cdot 3^2 \cdot 11 \]
Chapter 9 Assessment Answer Key

Form 3
Page 78

1. \(3^2x^2y\)

2. 91

3. prime

4. composite

5. \(-15x^5\)

6. \(4r^5t^3\)

7. \(\frac{a}{b^4}\)

8. 4m

9. \(6^{-2}\)

10. \(\frac{1}{243}\)

11. 0.0001057

12. \(5 \times 10^2 \text{ s}\)

13. \(3.0 \times 10^{-4}, 3.03 \times 10^{-4},
3.13 \times 10^{-4}, 0.00303,
0.0313\)

Page 79

14. \(n^{200}\)

15. \(256a^{84}\)

16. Linear; rate of change is constant.

17. Nonlinear; graph is not a straight line.

18. [Graph of a linear equation]

19. [Graph of a nonlinear equation]

20. [Graph of a nonlinear equation]

2, 3, 5, 7, 11, 13, 17, 19, 23,
29, 31, 37, 41, 43, and 47
## Chapter 9 Assessment Answer Key

### Page 80, Extended-Response Test

**Scoring Rubric**

<table>
<thead>
<tr>
<th>Score</th>
<th>General Description</th>
<th>Specific Criteria</th>
</tr>
</thead>
</table>
| **4** | Superior            | • Shows thorough understanding of the concepts factors, exponent, prime, composite, linear, and nonlinear.  
• Computations are correct.  
• Written explanations are exemplary.  
• Graph is accurate.  
• Goes beyond requirements of some or all problems. |
| **3** | Satisfactory        | • Shows an understanding of the concepts factors, exponent, prime, composite, linear, and nonlinear.  
• Computations to identify prime numbers are mostly correct.  
• Written explanations are effective.  
• Graph is mostly accurate.  
• Satisfies all requirements of problems. |
| **2** | Nearly Satisfactory | • Shows an understanding of most of the concepts factors, exponent, prime, composite, linear, and nonlinear.  
• Computations to identify prime numbers are mostly correct.  
• Written explanations are satisfactory.  
• Graph is mostly accurate.  
• Satisfies most requirements of the problems. |
| **1** | Nearly Unsatisfactory | • Final computation is correct.  
• No written explanations or work is shown to substantiate the final computation.  
• Graph may be accurate but not complete.  
• Satisfies minimal requirements of some of the problems. |
| **0** | Unsatisfactory      | • Shows little or no understanding of most of the concepts factors, exponent, prime, composite, linear, and nonlinear.  
• Computations to identify prime numbers are incorrect.  
• Written explanations are not satisfactory.  
• Graph is not accurate.  
• Does not satisfy requirements of problems. |
Chapter 9 Assessment Answer Key

Page 80, Extended-Response Test
Sample Answers

In addition to the scoring rubric found on page A36, the following sample answers may be used as guidance in evaluating open-ended assessment items.

1. Sample answer: 50 = 3 + 47, 74 = 13 + 61, 98 = 37 + 61

2a. True; any number divided by itself is 1.

2b. False; it is true for x = 0:
\[
\frac{0}{0 - 1} = \frac{0}{-1} = 0, \text{ but not true for } x = 20: \frac{2}{2 - 1} = \frac{2}{1} \neq 0.
\]

3a. 3a means 3 \cdot a or \( a + a + a \).  
\( a^3 \) means \( a \cdot a \cdot a \).

3b. \(-3b\) means \(-3 \cdot b\) or \(-\left(b + b + b\right)\).
\( b^{-3} \) means \( \frac{1}{b^3} \) or \( \frac{1}{b \cdot b \cdot b} \).

4a–b. Nonlinear; the graphed function is not a straight line.

4b. \( x^3 + 1 \)
Chapter 9 Assessment Answer Key

Standardized Test Practice

Page 81

1. ● ○ ○ ○
2. ○ ○ ● ○
3. ● ○ ○ ○
4. ○ ○ ● ○
5. ○ ○ ● ○
6. ○ ● ○ ○
7. ○ ● ○ ○
8. ○ ● ○ ○
9. ● ○ ○ ○
10. ○ ● ● ○

Page 82

11. ○ ● ○ ○
12. ● ○ ○ ○
13. ○ ● ● ○
14. ○ ● ○ ○
15. ○ ○ ○ ●
16. ○ ● ○ ○
17. ○ ● ○ ○
18. ● ○ ○ ○
19. [Diagram]
20. [Diagram]
Chapter 9 Assessment Answer Key

Standardized Test Practice

Page 83

21. \( D = \{2.3, 5, 4.6\}; \)
   \( R = \{4, 3.2, 3.3\} \)

22. \(-22m\)

23. \(32x\)

24. \(4\)

25. \(2\)

26. \(IV\)

27. \(\frac{x}{-9} = 18; -162\)

28. \(1296\)

29. \(960\)

30. \(2 \cdot 3 \cdot 13\)

31. \(7\) bags

32. \(2.85 \times 10^9\)

33. \(-1\)

34. \(y = 3x + 7\)

35a. \(t = 4n + 16\)

35b. \(40\)
Chapter 9 Assessment Answer Key

Unit 3 Test
Page 84

As the altitude increases, the temperature decreases.
1. Yes, it is a function.

2. 102
   Sample answer: (2, 1), (0, 0)

3. (-2, 0); (0, 2);

4. \(\frac{1}{2}\)

5. \(-\frac{2}{5}\) °F/mph

6. \(y = 6x - 3\)

Page 85

10. \(3^{12}\)
11. 192
12. 44
13. \(2^4 \cdot 3^2\)
14. \(21m^4n^2\)
15. \(a^{15}\)
16. \(\frac{1}{5^7}\)
17. \(2.54005 \times 10^7\)
   \(1.4 \times 10^{-3}; 4.1 \times 10^{-3}; 0.041; 4100; 41,000;\)
18. \(4.1 \times 10^9\)
19. \(-64t^9\)
   Nonlinear; the equation cannot be written as \(y = mx + b\).
20. 

21. \(y = x^2 - 2\)

22. \(y = -2x^2 + 1\)

23. \(y = 3^x - 3\)